Mini-Quiz Mar. 17

Your name:_____

- This quiz is **closed book**. Total time is 25 minutes.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	8		
2	6		
3	6		
Total	20		

Problem 1 (8 points).
Starting with some number of 4-cent and 7-cent stamps on the table, there are two ways to change the stamps:

(i) Add *one* 4-cent stamp, or

(ii) remove *two* 4-cent AND *two* 7-cent stamps (when this is possible).

(a) Let A be the number of 4-cent stamps; and B be the number of 7-cent stamps. The chart below indicates properties of some derived variables; fill it in.

derived variables:	B	$4A + 7B$	$\text{rem}(B, 2)$	$\text{rem}(4 * A + 7 * B, 2)$
weakly increasing				
strictly increasing				
weakly decreasing				
strictly increasing				
constant				

(b) Circle the properties below that are preserved invariants:

1. The number of 7-cent stamps (B) must be even.
2. The number of 7-cent stamps (B) must be greater than 0.
3. The total cost of stamps ($4 * A + 7 * B$) must be odd.
4. $4A > 7B$.

(c) Using the Invariant Principle, show that it is impossible to have stamps with a total value of exactly 90 cents on the table when we start with exactly 211 7-cent stamps. (You may use without proof the preserved invariance of some of the properties from part (b).)

Problem 2 (6 points).
Covering edges were introduced in class problem: if a and b are distinct vertices of a digraph, then a is said to *cover* b if there is an edge from a to b and every path from a to b traverses this edge. If a covers b, the edge from a to b is called a *covering edge*.

Let D be a finite directed acyclic graph (DAG).

(a) If there is a path in D from a vertex, u, to vertex, v, explain why there must be a *longest* path from u to v.

(b) Give a proof of the following claim from the class problem:
Claim. *If there is a path in D from a vertex, u, to vertex, v, then there is a path from u to v that only traverses covering edges.*

(c) Show that the Claim fails for the finite digraph, F, with three vertices and edges from every vertex to every other vertex. *Hint:* What are the covering edges of F?

Problem 3 (6 points).
Let G be a connected simple graph. Prove that if an edge in a connected graph is not traversed by any simple cycle, then it is a cut edge.[1]

[1] A simple cycle is a subgraph of G isomorphic to the cycle graph C_n for $n \geq 3$. An edge is a *cut-edge* when removing the edge disconnects the graph.

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