Mini-Quiz Mar. 3

Your name: ____

- This quiz is **closed book**. Total time is 25 minutes.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	6		
2	7		
3	7		
Total	20		

Problem 1 (6 points).
Each year, Santa's reindeer hold "Reindeer Games," from which Rudolph is pointedly excluded. The Games consist of a sequence of matches between pairs of reindeer. Draws are not possible.

On Christmas Eve, Santa produces a rank list of all his reindeer. If reindeer p lost a match to reindeer q, then p appears below q in Santa's ranking, but if he has any choice because of unplayed matches, Santa can give higher rank to the reindeer he likes better. To prevent confusion, two reindeer may not play a match if either outcome would lead to a cycle of reindeer, where each lost to the next.

Though it is only March, the 2010 Reindeer Games have already begun (punctuality is key at the north pole). We can describe the results so far with a binary relation, L, on the set of reindeer, where pLq means that reindeer p lost in a match to reindeer q. Let $\prec$ be the corresponding *indirectly lost* relation, where reindeer p indirectly lost to reindeer q when p lost a match with q, or when p lost to a reindeer who lost to q, or when p lost to a reindeer who lost to a reindeer who lost to q, etc. Note that the "indirectly lost" relation, $\prec$, is a partial order.

On the following page you'll find a list of terms and a sequence of statements. Add the appropriate term to each statement.

Terms

a strict partial order	a weak partial order	a total order
comparable elements	incomparable elements	a chain
an antichain	a maximal antichain	a topological sort
a minimum element	a minimal element	
a maximum element	a maximal element	

Statements

(a) A reindeer who is unbeaten so far is

____ of the partial order $\prec$.

(b) A reindeer who has lost every match so far is

____ of the partial order $\prec$.

(c) Two reindeer can *not* play a match if they are

____ of $\prec$.

(d) A reindeer assured of first place in Santa's ranking is

____ of $\prec$.

(e) A sequence of reindeer which *must* appear in the same order in Santa's rank list is

____.

(f) A set of reindeer such that any two could still play a match is

____.

(g) The fact that no reindeer loses a match to himself implies that $\prec$ is

____.

(h) Santa's final ranking of his reindeer on Christmas Eve must be

____ of $\prec$.

(i) No more matches are possible if and only if $\prec$ is

____.

Problem 2 (**7 points**).
Prove *by induction* that every exact amount of postage of 12 cents or more can be formed using 3 and 7 cent stamps. In particular, clearly identify

- the induction variable,
- the induction hypothesis,
- the base case(s), and
- the inductive step.

Problem 3 **(7 points).**
Let $[\mathbb{N} \to \{1, 2, 3\}]$ be the set of infinite sequences containing only the numbers 1, 2, and 3. For example, some sequences of this kind are:

$$(1, 1, 1, 1...),$$
$$(2, 2, 2, 2...),$$
$$(3, 2, 1, 3...).$$

Prove that $[\mathbb{N} \to \{1, 2, 3\}]$ is uncountable.

Hint: One approach is to define a surjective function from $[\mathbb{N} \to \{1, 2, 3\}]$ to the power set $\mathcal{P}(\mathbb{N})$.