# Problem Set 12

*Due*: May 7

**Reading:** This pset covers Notes Ch. 18, **skipping** §§18.3.5–18.3.7 and Ch.**??**. Additional reading for class May 7–12 is specified in online Tutor Problems 13.

**Problem 1.**
Outside of their hum-drum duties as 6.042 LAs, Oscar is trying to learn to levitate using only intense concentration and Liz is trying to become the world champion flaming torch juggler. Suppose that Oscar's probability of success is $1/6$, Liz's chance of success is $1/4$, and these two events are independent.

**(a)** If at least one of them succeeds, what is the probability that Oscar learns to levitate?

**(b)** If at most one of them succeeds, what is the probability that Liz becomes the world flaming torch juggler champion?

**(c)** If exactly one of them succeeds, what is the probability that it is Oscar?

**Problem 2.**
Here are seven propositions:

$$\begin{array}{rcrcr}
x_1 & \vee & x_3 & \vee & \neg x_7 \\
\neg x_5 & \vee & x_6 & \vee & x_7 \\
x_2 & \vee & \neg x_4 & \vee & x_6 \\
\neg x_4 & \vee & x_5 & \vee & \neg x_7 \\
x_3 & \vee & \neg x_5 & \vee & \neg x_8 \\
x_9 & \vee & \neg x_8 & \vee & x_2 \\
\neg x_3 & \vee & x_9 & \vee & x_4
\end{array}$$

Note that:

1. Each proposition is the OR of three terms of the form $x_i$ or the form $\neg x_i$.
2. The variables in the three terms in each proposition are all different.

Suppose that we assign true/false values to the variables $x_1, \ldots, x_9$ independently and with equal probability.

**(a)** What is the expected number of true propositions?

**(b)** Use your answer to prove that there exists an assignment to the variables that makes *all* of the propositions true.

**Problem 3.**
Each 6.042 final exam will be graded according to a rigorous procedure:

- With probability $\frac{4}{7}$ the exam is graded by a *TA*,with probability $\frac{2}{7}$ it is graded by a *lecturer*, and with probability $\frac{1}{7}$, it is accidentally dropped behind the radiator and arbitrarily given a score of 84.
- *TAs* score an exam by scoring each problem individually and then taking the sum.
  - There are ten true/false questions worth 2 points each. For each, full credit is given with probability $\frac{3}{4}$, and no credit is given with probability $\frac{1}{4}$.
  - There are four questions worth 15 points each. For each, the score is determined by rolling two fair dice, summing the results, and adding 3.
  - The single 20 point question is awarded either 12 or 18 points with equal probability.
- *Lecturers* score an exam by rolling a fair die twice, multiplying the results, and then adding a "general impression"score.
  - With probability $\frac{4}{10}$, the general impression score is 40.
  - With probability $\frac{3}{10}$, the general impression score is 50.
  - With probability $\frac{3}{10}$, the general impression score is 60.

Assume all random choices during the grading process are independent.

**(a)** What is the expected score on an exam graded by a TA?

**(b)** What is the expected score on an exam graded by a lecturer?

**(c)** What is the expected score on a 6.042 final exam?

**Problem 4.**
The most common way to build a local area network nowadays is an Ethernet: basically, a single wire to which we can attach any number of computers. The communication protocol is quite simple: anyone who wants to talk to another computer broadcasts a message on the wire, hoping the other computer will hear it. The problem is that if *more* than one computer broadcasts at once, a *collision* occurs that garbles all messages we are trying to send. The transmission only works if *exactly one* machine broadcasts at one time.

Let's consider a simple example. There are $n$ machines connected by an ethernet, and each wants to broadcast a message. We can imagine time divided into a sequence of intervals, each of which is long enough for one message broadcast.

Suppose each computer flips an independent coin, and decides to broadcast with probability $p$.

**(a)** What is the probability that exactly one message gets through in a given interval? *Hint:* Consider the event $A_i$ that machine $i$ transmits but no other does.

**(b)** What is the expected time it takes for machine $i$ to get a message through?

# Student's Solutions to Problem Set 12

| | |
|---|---|
| **Your name:** | |
| **Due date:** | May 7 |
| **Submission date:** | |
| **Circle your TA/LA:** | Megumi    Tom    Richard    Eli |

**Collaboration statement:** Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:

   got help from:[1]

   and referred to:[2]

**DO NOT WRITE BELOW THIS LINE**

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total | |



[1]People other than course staff.
[2]Give citations to texts and material other than the Spring '10 course materials.