Problem Set 1

Due: February 12

Reading: Chapters 1, *What is a Proof?*; 2, *The Well Ordering Principle*; 3, *Propositional Formulas*.

These assigned readings do not include the Problem sections. (Many of the problems in the text will appear as class or homework problems.)

Reminder: Comments on the reading using the *NB online annotation system* are due at times indicated in the online tutor problem set TP.2. Reading Comments count for 5% of the final grade.

Problem 1.
A recent class problem proved that there were irrational numbers a, b such that a^b was rational. Unfortunately, that proof was *nonconstructive*: it didn't reveal a specific pair, a, b, with this property. But in fact, it's easy to do this: let $a ::= \sqrt{2}$ and $b ::= 2\log_2 3$.

We know $\sqrt{2}$ is irrational, and obviously $a^b = 3$. Finish the proof that this a, b pair works, by showing that $2\log_2 3$ is irrational.

Problem 2.
For $n = 40$, the value of polynomial $p(n) ::= n^2 + n + 41$ is not prime, as noted in Chapter 1 of the Course Text. But we could have predicted based on general principles that no nonconstant polynomial, $q(n)$, with integer coefficients can map each nonnegative integer into a prime number. Prove it.

Hint: Let $c ::= q(0)$ be the constant term of q. Consider two cases: c is not prime, and c is prime. In the second case, note that $q(cn)$ is a multiple of c for all $n \in \mathbb{Z}$. You may assume the familiar fact that the magnitude (absolute value) of any nonconstant polynomial, $q(n)$, grows unboundedly as n grows.

Problem 3.
Describe a simple recursive procedure which, given a positive integer argument, n, produces a truth table whose rows are all the assignments of truth values to n propositional variables. For example, for $n = 2$, the table might look like:

T	T
T	F
F	T
F	F

Your description can be in English, or a simple program in some familiar language (say Scheme or Java), but if you do write a program, be sure to include some sample output.

Problem 4.
Prove that the propositional formulas

$$P \text{ OR } Q \text{ OR } R$$

and

$$(P \text{ AND NOT } Q) \text{ OR } (Q \text{ AND NOT } R) \text{ OR } (R \text{ AND NOT } P) \text{ OR } (P \text{ AND } Q \text{ AND } R).$$

are equivalent.

Problem 5.
Use the Well Ordering Principle to prove that

$$n \leq 3^{n/3} \tag{1}$$

for every nonnegative integer, n.

Hint: Verify (1) for $n \leq 4$ by explicit calculation.

Student's Solutions to Problem Set 1

Your name:

Due date: February 12

Submission date:

Circle your TA/LA: Megumi Tom Richard Eli

Collaboration statement: Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:

   got help from:[1]

   and referred to:[2]

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	
2	
3	
4	
5	
Total	

[1]People other than course staff.
[2]Give citations to texts and material other than the Spring '10 course materials.