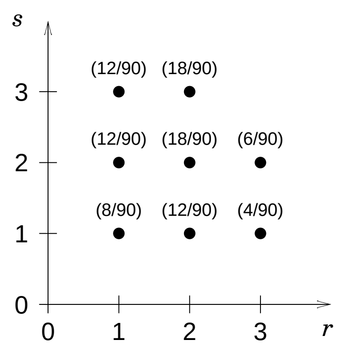
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial 3
March 2-3, 2006

1. 

The joint PMF for random variables R and S is depicted in the sketch as follows: A point at (r, s) is labeled with $\mathbf{P}(R = r, S = s)$ for all pairs with positive probability. Let A denote the event $\{S \neq 3\}$.

   (a) Prepare neat, fully-labeled sketches of $p_S(s)$ and $p_{S|A}(s)$.

   (b) Let $Y = R - S$. Prepare a neat, fully-labeled sketch of $p_{R,Y}(r, y)$.

   (c) Define the random variable $X = R + S$. Prepare a neat, fully-labeled plot of $p_{X|A}(x)$.

2. Chuck will go shopping for probability books for K hours. Here, K is a random variable and is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops. We are told that

$$p_{N|K}(n \mid k) = \frac{1}{k}, \qquad \text{for } n = 1, \ldots, k.$$

   (a) Find the joint PMF of K and N.

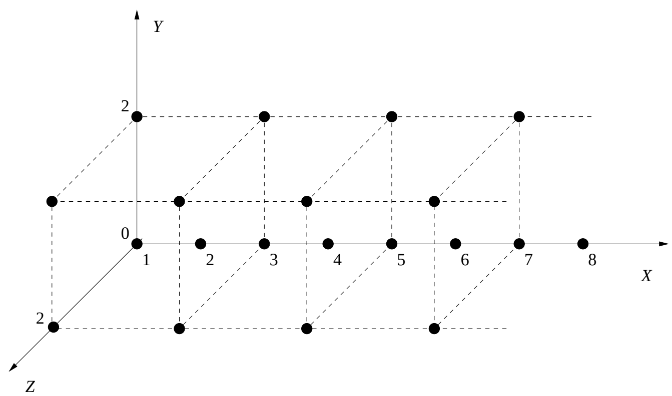
   (b) Find the marginal PMF of N.

   (c) Find the conditional PMF of K given that $N = 2$.

   (d) We are now told that he bought at least 2 but no more than 3 books. Find the conditional mean and variance of K, given this piece of information.

   (e) The cost of each book is a random variable with mean 3. What is the expected value of his total expenditure? *Hint:* Condition on events $N = 1, \ldots, N = 4$ and use the total expectation theorem.

3. Consider three random variables X, Y, and Z, associated with the same experiment. The random variable X is geometric with parameter p. If X is even, then Y and Z are equal to zero. If X is odd, (Y, Z) is uniformly distributed on the set $S = \{(0, 0), (0, 2), (2, 0), (2, 2)\}$. The figure below shows all the possible values for the triple (X, Y, Z) that have $X \leq 8$. (Note that the X axis starts at 1 and that a complete figure would extend indefinitely to the right.)



(a) Find the joint PMF $p_{X,Y,Z}(x,\ y,\ z)$

(b) Answer with "yes" or "no" and one sentence of explanation:

(i) Are Y and Z independent?

(ii) Given that $Z = 2$, are X and Y independent?

(iii) Given that $Z = 0$, are X and Y independent?

(iv) Given that $Z = 2$, are X and Z independent?

(c) Find $\text{var}((Y + Z) \mid X = 5)$.