

6.041/6.431 Spring 2006 Quiz 2
Wednesday, March 19, 7:30-9:30 PM.

DO NOT TURN THIS PAGE OVER UNTIL
YOU ARE TOLD TO DO SO

Name: _____

Recitation Instructor: _____

TA: _____

Question	Score	Out of
0		3
1		30
2		25
3		42
Your Grade		100

- You have 120 minutes to complete the quiz.
- At the end of the quiz period you will turn in this quiz packet, and 2 blue books. Question 1 will be answered in the quiz packet, while question 2 and 3 will be answered in their own respective blue books.
- Problem 1 is True-False; no partial credit is given.
- For Problems 2 and 3, you should concisely indicate your reasoning and show all relevant work. Grades will be based on our judgment of your level of understanding as reflected by what you have written.
- Unless otherwise stated, you may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. However you should reduce expressions as much as possible to receive full credit. Solutions in class problem sets, and past quizzes serve as examples.
- This is a closed-book exam except for 2 double-sided, handwritten, 8.5 by 11 formula sheet.
- Calculators are allowed.
- Be neat! If we can't read it, we can't grade it.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Problem 0: (3 points)

Write your name, your recitation instructor's name, and TA's name on the cover of the quiz booklet and your 2 blue books. Include the question number on the cover of the blue book.

Problem 1: (30 points)

Each of the following statements is either True or False. There will be **no partial credit** given for the True False questions, thus any explanations will not be graded. Please **clearly** indicate True or False in the below, ambiguous marks will receive zero credit. All parts have equal weight.

- (a) X and Y are independent random variables. X is uniformly distributed on the interval $[-2, 2]$, while Y is uniformly distributed on the interval $[-1, 5]$. If $Z = X + Y$, then $f_Z(3) = 1/6$.

True False

- (b) If X is a Gaussian random variable with zero mean and variance equal to 1, then the density function of $Z = |X|$ is equal to $2f_X(z), z \geq 0$.

True False

- (c) The sum of a random number of independent Gaussian random variables with zero mean and unit variance results in a Gaussian random variable regardless of the distribution of N (the number of variables in the sum).

True False

- (d) If X and Y are independent random variables, both exponentially distributed with parameters λ_1 and λ_2 respectively. Then the the random variable $Z = \min\{X, Y\}$ is also exponentially distributed.

True False

- (e) Let the transform associated with a random variable X be

$$M_X(s) = \left(\frac{e^s}{1-s} \right)^{15}.$$

Then $\mathbf{E}[X]$ is equal to 30.

True False

The next set of questions are concerned with two independent random variables: Y is normal with mean 0 and variance 1, and X is uniform between $[0, 1]$. $Z = X + Y$.

- (f) The conditional density of Z given X , $f_{Z|X}(z|x)$, is normal with mean x and variance 1.

True False

- (g) $\text{var}(Z) = 2$.

True False

- (h) $\mathbf{E}[X | Z = -1] = -1$.

True False

- (i) $\text{cov}(X, Z) = \text{var}(X)$

True False

- (j) $Z = \mathbf{E}[X | Z] + \mathbf{E}[Y | Z]$

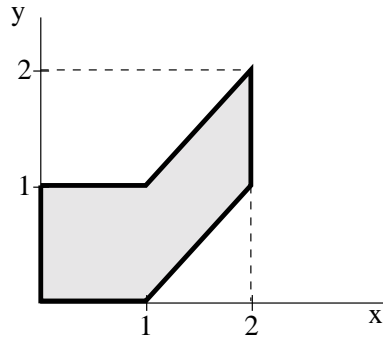
True False

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Problem 2: (25 points)

Please write all work for Problem 2 in your **first blue book**. No work recorded below will be graded. All parts have approximately the same weight.

The continuous random variables X and Y have a joint pdf given by



$$f_{X,Y}(x,y) = \begin{cases} c, & \text{if } (x,y) \text{ belongs to the shaded region;} \\ 0, & \text{otherwise.} \end{cases}$$

In class we have shown the minimum least squares estimate of Y is given by $\mathbf{E}[Y|X = x]$

- (a) Find the least squares estimate of Y given that $X = x$, for all possible values of x . For full credit write the functional form, as opposed to a graph.
- (b) Let $g(x)$ be the estimate from part (a). Find $\mathbf{E}[g(X)]$ and $\text{var}(g(X))$.
- (c) Find the mean square error $\mathbf{E}[(Y - g(X))^2]$. Is it the same as $\mathbf{E}[\text{var}(Y|X)]$?
- (d) Find $\text{var}(Y)$.

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Problem 3: (42 points)

Please write all work for Problem 3 in your **second blue book**. No work recorded below will be graded. All parts have approximately the same weight.

Each year, a publisher sends Professor MD a random number of text books to review. The number of books Professor MD receives each year can be modeled as a Poisson random variable N , with mean μ . Each book contains a random number of typos, where the number of typos in one book can be modeled as a Poisson random variable with mean λ . Let B_i denote the number of typos in book i . Assume N is independent of B_i for all i , and B_i is independent of B_j for all $i \neq j$. Professor MD is an expert in the field of typo identification, but even experts aren't perfect. Assume Professor MD finds any existing typo with probability p , and that this is independent of finding any other typos and also independent of N and B_i .

The publisher offers Professor MD two different annual salary options for reviewing the text books. The two options are:

Option 1: 1 dollar for each typo found.

Option 2: 1 dollar for each book where at least one typo is found.

Let X_i be the amount of money Professor MD receives for book i , and let T be the total amount of money Professor MD receives in any given year.

- (a) Find and correctly state the PMF of X_i under option 1. For full credit reduce this expression to a well known PMF. What's the name of this PMF?
- (b) Find $M_T(s)$ under option 1.
- (c) Find $\mathbf{P}(T = 2)$ under option 1.
- (d) Find $\mathbf{E}[T]$ under option 1.
- (e) Find $\text{var}(T)$ under option 1.
- (f) Find and correctly state the PMF of X_i under option 2. For full credit reduce this expression to a well known PMF. What's the name of this PMF?
- (g) Find $\mathbf{E}[T]$ under option 2. Hint: Fully reduce your answer in (f) before attempting.