Recitation 18
November 9, 2010

1. There are n fish in a lake, some of which are green and the rest blue. Each day, Helen catches 1 fish. She is equally likely to catch any one of the n fish in the lake. She throws back all the fish, but paints each green fish blue before throwing it back in. Let G_i denote the event that there are i green fish left in the lake.

   (a) Show how to model this fishing exercise as a Markov chain, where $\{G_i\}$ are the states. Explain why your model satisfies the Markov property.

   (b) Find the transition probabilities $\{p_{ij}\}$.

   (c) List the transient and the recurrent states.

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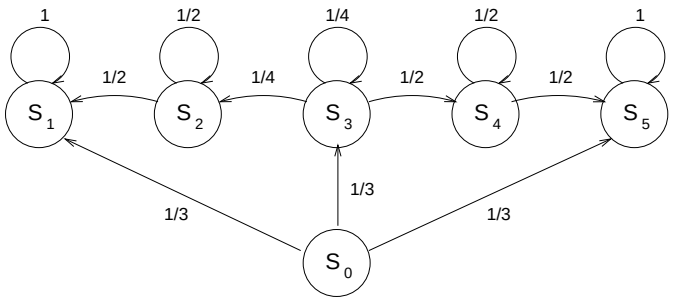
Drake, Fundamentals of Applied Probability Theory, Problem 5.02.

3. Consider the following Markov chain, with states labelled from $s_0, s_1, \ldots, s_5$:



Given that the above process is in state s_0 just before the first trial, determine by inspection the probability that:

   (a) The process enters s_2 for the first time as the result of the kth trial.

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(b) The process never enters s_4.

(c) The process enters s_2 and then leaves s_2 on the next trial.

(d) The process enters s_1 for the first time on the third trial.

(e) The process is in state s_3 immediately after the nth trial.

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6.041 / 6.431 Probabilistic Systems Analysis and Applied Probability

Fall 2010

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