Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

Recitation 11
October 14, 2010

1. Let X be a discrete random variable that takes the values 1 with probability p and -1 with probability $1-p$. Let Y be a continuous random variable independent of X with the Laplacian (two-sided exponential) distribution

$$f_Y(y) \;=\; \tfrac{1}{2}\lambda e^{-\lambda|y|},$$

and let $Z = X+Y$. Find $\mathbf{P}(X=1 \mid Z=z)$. Check that the expression obtained makes sense for $p \to 0^+$, $p \to 1^-$, $\lambda \to 0^+$, and $\lambda \to \infty$.

2. Let Q be a continuous random variable with PDF

$$f_Q(q) \;=\; \begin{cases} 6q(1-q), & \text{if } 0 \le q \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

This Q represents the probability of success of a Bernoulli random variable X, i.e.,

$$\mathbf{P}(X=1 \mid Q=q) \;=\; q.$$

Find $f_{Q|X}(q|x)$ for $x \in \{0,1\}$ and all q.

3. Let X have the normal distribution with mean 0 and variance 1, i.e.,

$$f_X(x) \;=\; \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Also, let $Y = g(X)$ where

$$g(t) \;=\; \begin{cases} -t, & \text{for } t \le 0; \\ \sqrt{t}, & \text{for } t > 0, \end{cases}$$

as shown to the right.

Find the probability density function of Y.