

Recitation 11
October 14, 2010

1. Let X be a discrete random variable that takes the values 1 with probability p and -1 with probability $1 - p$. Let Y be a continuous random variable independent of X with the Laplacian (two-sided exponential) distribution

$$f_Y(y) = \frac{1}{2}\lambda e^{-\lambda|y|},$$

and let $Z = X + Y$. Find $\mathbf{P}(X = 1 \mid Z = z)$. Check that the expression obtained makes sense for $p \rightarrow 0^+$, $p \rightarrow 1^-$, $\lambda \rightarrow 0^+$, and $\lambda \rightarrow \infty$.

2. Let Q be a continuous random variable with PDF

$$f_Q(q) = \begin{cases} 6q(1 - q), & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

This Q represents the probability of success of a Bernoulli random variable X , i.e.,

$$\mathbf{P}(X = 1 \mid Q = q) = q.$$

Find $f_{Q|X}(q|x)$ for $x \in \{0, 1\}$ and all q .

3. Let X have the normal distribution with mean 0 and variance 1, i.e.,

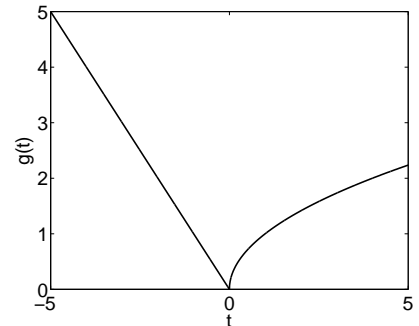
$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Also, let $Y = g(X)$ where

$$g(t) = \begin{cases} -t, & \text{for } t \leq 0; \\ \sqrt{t}, & \text{for } t > 0, \end{cases}$$

as shown to the right.

Find the probability density function of Y .



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