Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

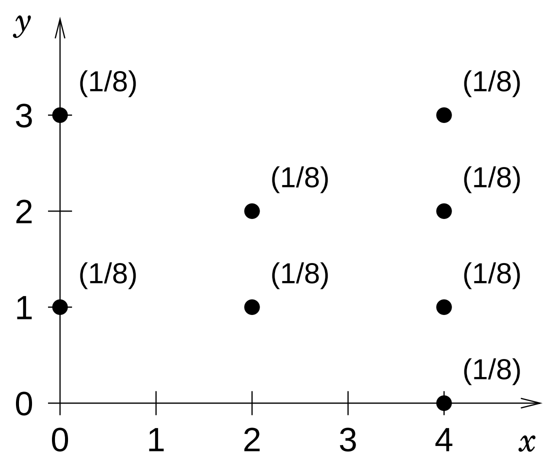
Recitation 6
September 28, 2010

1. Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables N and K are defined by

   - $N =$ the result of the die roll
   - $K =$ the total number of heads resulting from the coin flips

   (a) Determine and sketch $p_N(n)$
   (b) Determine and tabulate $p_{N,K}(n,k)$
   (c) Determine and sketch $p_{K|N}(k \mid 2)$
   (d) Determine and sketch $p_{N|K}(n \mid 2)$

2. Consider an outcome space comprising eight equally likely event points, as shown below:



   (a) Which value(s) of x *maximize(s)* $\mathbf{E}[Y \mid X = x]$?
   (b) Which value(s) of y *maximize(s)* $\text{var}(X \mid Y = y)$?
   (c) Let $R = \min(X, Y)$. Prepare a neat, fully labeled sketch of $p_R(r)$,
   (d) Let A denote the event $X^2 \geq Y$. Determine numerical values for the quantities $\mathbf{E}[XY]$ and $\mathbf{E}[XY \mid A]$.

3. **Example 2.17. Variance of the geometric distribution.** You write a software program over and over, and each time there is probability p that it works correctly, independent of previous attempts. What is the variance of X, the number of tries until the program works correctly?

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