6.01: Introduction to EECS I

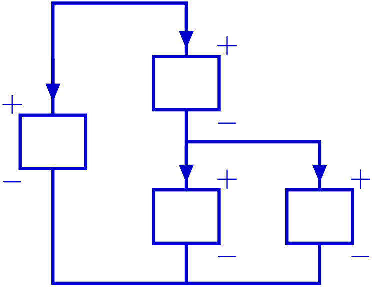
Op-Amps

March 29, 2011

Last Time: The Circuit Abstraction

Circuits represent systems as connections of elements

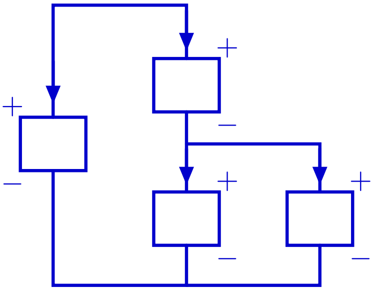
- through which currents (through variables) flow and
- across which voltages (across variables) develop.



Last Time: Analyzing Circuits

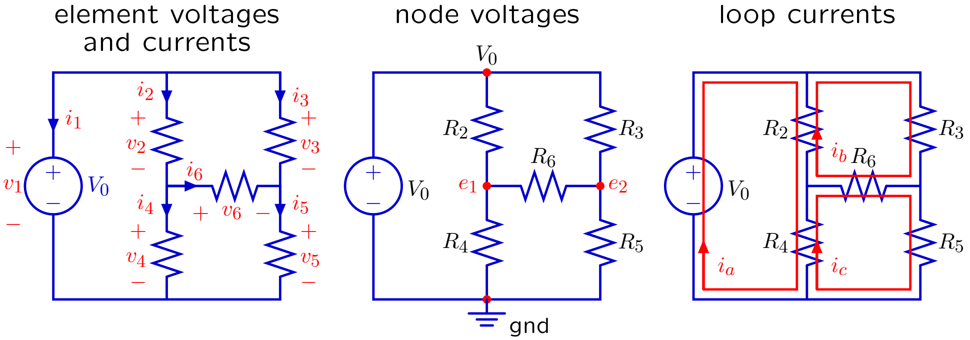
Circuits are analyzed by combining three types of equations.

- KVL: sum of voltages around any closed path is zero.
- KCL: sum of currents out of any closed surface is zero.
- Element (constitutive) equations
  - resistor: $V = IR$
  - voltage source: $V = V_0$
  - current source: $I = I_0$

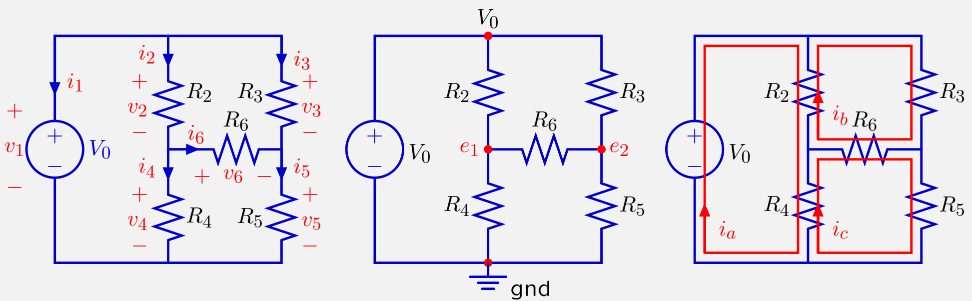


Last Time: Analyzing Circuits

Many KVL and KCL equations are redundant. We looked at three methods to systematically identify a linearly independent set.



Check Yourself



How many of the following are true?

1. $v_1 = v_2 + v_6 + v_5$
2. $v_6 = e_1 - e_2$
3. $i_6 = (e_1 - e_2)/R_6$
4. $i_6 = i_b - i_c$
5. $v_6 = (i_b - i_c)R_6$

Check Yourself

How many of the following are true? 3

1. $v_1 = v_2 + v_6 + v_5$ √
2. $v_6 = e_1 - e_2$ √
3. $i_6 = (e_1 - e_2)/R_6$ √
4. $i_6 = i_b - i_c$ ✗ $i_6 = i_c - i_b$
5. $v_6 = (i_b - i_c)R_6$ ✗ $v_6 = (i_c - i_b)R_6$

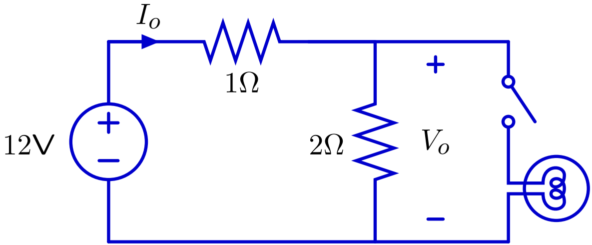
Node Voltages with Component Currents

We will study a variation of the node method (NVCC) in software lab today.

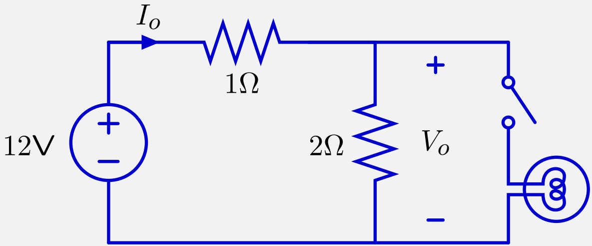
Interaction of Circuit Elements

Circuit design is complicated by interactions among the elements. Adding an element changes voltages & currents **throughout** circuit.

Example: closing a switch is equivalent to adding a new element.



Check Yourself

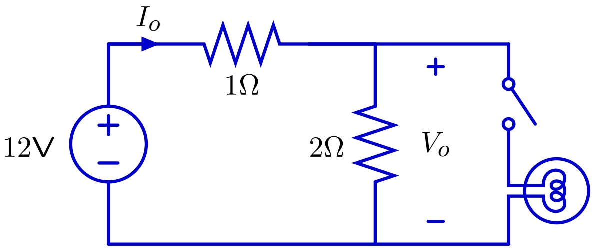


How does closing the switch affect V_o and I_o?

1. V_o decreases, I_o decreases
2. V_o decreases, I_o increases
3. V_o increases, I_o decreases
4. V_o increases, I_o increases
5. could be any of above, depending on bulb resistance

Check Yourself

Start by computing V_o and I_o when the switch is open.



Calculate V_o using voltage divider relation:

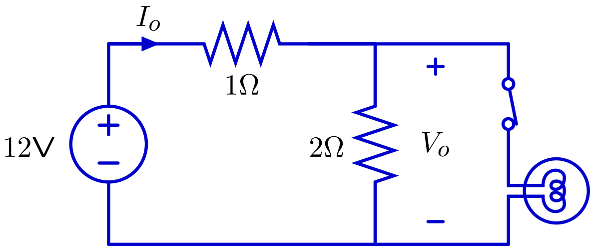
$$V_o = \frac{2\Omega}{1\Omega + 2\Omega} 12\text{V} = 8\text{V}$$

Calculate I_o by lumping resistors into series equivalent:

$$I_o = \frac{12\text{V}}{1\Omega + 2\Omega} = 4\text{A}$$

Check Yourself

Now compute V_o and I_o when the switch is closed.

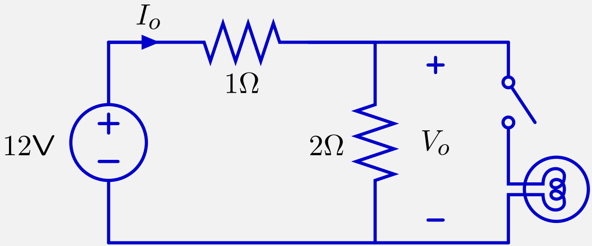


Assume the light bulb can be represented by a resistor R $(0 < R < \infty)$. Then R is in parallel with the 2Ω resistor.

$$V_o = \frac{2\Omega||R}{1\Omega + 2\Omega||R}\,12\text{V} = \frac{\frac{2\Omega\times R}{2\Omega+R}}{1\Omega + \frac{2\Omega\times R}{2\Omega+R}}\,12\text{V} = \frac{2R}{2\Omega+3R}\,12\text{V} \le 8\text{V}$$

$$I_o = \frac{12\text{V}}{1\Omega + 2\Omega||R} = \frac{12\text{V}}{1\Omega + \frac{2\Omega\times R}{2\Omega+R}} = \frac{2\Omega+R}{2\Omega+3R}\,12\text{A} \ge 4\text{A}$$

Check Yourself

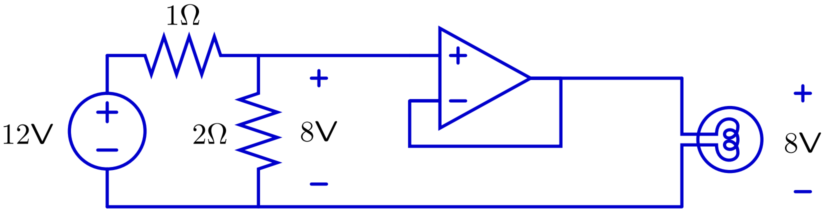


How does closing the switch affect V_o and I_o? 2

1. V_o decreases, I_o decreases
2. V_o decreases, I_o increases
3. V_o increases, I_o decreases
4. V_o increases, I_o increases
5. could be any of above, depending on bulb resistance

Buffering with Op-Amps

Interactions between elements can be reduced (or eliminated) by using an op-amp as a **buffer**.



This op-amp circuit produces an output voltage equal to its input voltage (8V) while having no effect on the left part of the circuit.

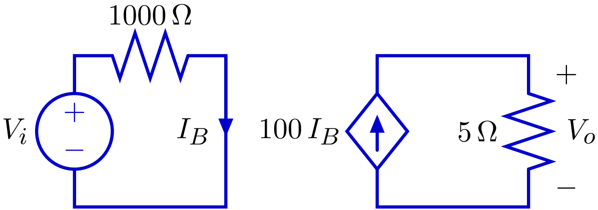
Today: how to analyze and design op-amp circuits

Dependent Sources

To analyze op-amps, we must introduce a new kind of element: a dependent source.

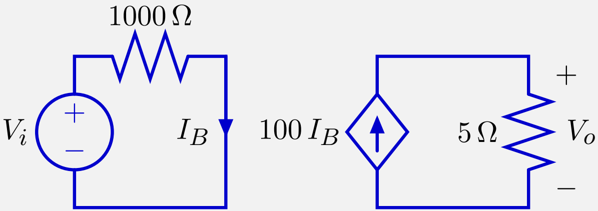
A dependent source generates a voltage or current whose value depends on another voltage or current.

Example: current-controlled current source



Check Yourself

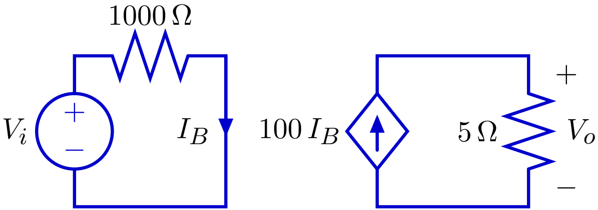
Find $\frac{V_o}{V_i}$.



1. 500
2. $\frac{1}{20}$
3. 1
4. $\frac{1}{2}$
5. none of the above

Check Yourself

Find $\frac{V_o}{V_i}$.

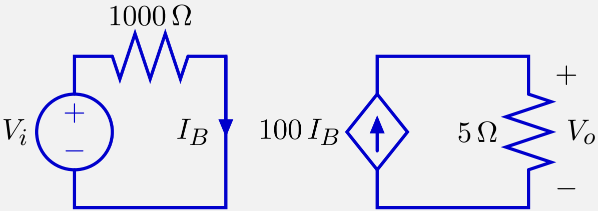


$$I_B = \frac{V_i}{1000\,\Omega}$$

$$V_o = 100\,I_B \times 5\,\Omega = 100\,\frac{V_i}{1000\,\Omega} \times 5\,\Omega = \frac{1}{2}V_i$$

Check Yourself

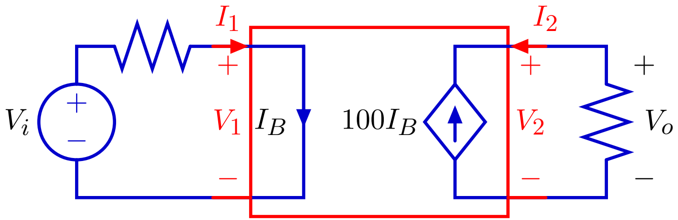
Find $\frac{V_o}{V_i}$. 4



1. 500
2. $\frac{1}{20}$
3. 1
4. $\frac{1}{2}$
5. none of the above

Dependent Sources

Dependent sources are two-ports: characterized by two equations.

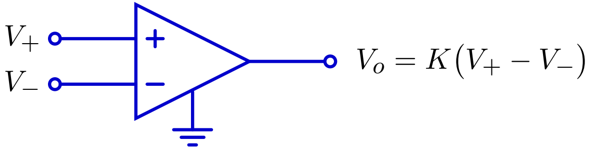


Here $V_1 = 0$ and $I_2 = -100\, I_1$.

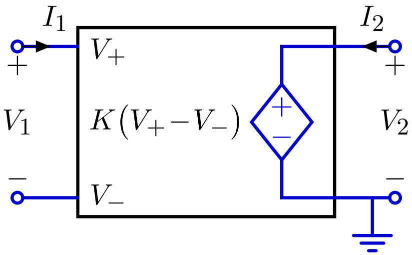
By contrast, one-ports (resistors, voltage sources, current sources) are characterized by a single equation.

Op-Amp

An op-amp (operational amplifier) can be represented by a voltage-controlled voltage source.



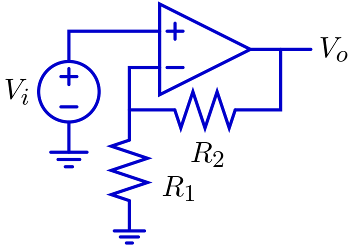
A voltage-controlled voltage source is a two-port.



$I_1 = 0$ and $V_2 = KV_1$ where K is large (typically $K > 10^5$).

Op-Amp: Analysis

Example. Find $\frac{V_o}{V_i}$ for the following circuit.



$$V_+ = V_i$$

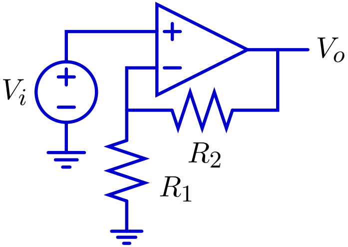
$$V_- = \frac{R_1}{R_1 + R_2} V_o$$

$$V_o = K\left(V_+ - V_-\right) = K\left(V_i - \frac{R_1}{R_1 + R_2} V_o\right)$$

$$\frac{V_o}{V_i} = \frac{K}{1 + \frac{KR_1}{R_1+R_2}} = \frac{K\left(R_1 + R_2\right)}{R_1 + R_2 + KR_1} \approx \frac{R_1 + R_2}{R_1} \quad \text{(if } K \text{ is large)}$$

Non-inverting Amplifier

For large K, this circuit implements a non-inverting amplifier.

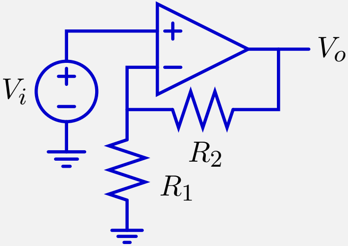


$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1} \geq 1$$

$$V_o \geq V_i$$

Check Yourself

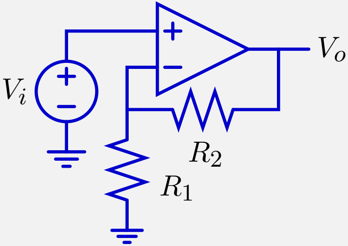
For which value(s) of R_1 and/or R_2 is $V_o = V_i$.



1. $R_1 \to \infty$
2. $R_2 = 0$
3. $R_1 \to \infty$ and $R_2 = 0$
4. all of the above
5. none of the above

Check Yourself

For which value(s) of R_1 and/or R_2 is $V_o = V_i$. 4

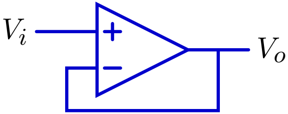


1. $R_1 \to \infty$
2. $R_2 = 0$
3. $R_1 \to \infty$ and $R_2 = 0$
4. all of the above — all are unity buffers
5. none of the above

The "Ideal" Op-Amp

As $K \rightarrow \infty$, the difference between V_+ and V_- goes to zero.

Example:



$$V_o = K\ \left(V_+ - V_-\right) = K\ \left(V_i - V_o\right)$$

$$V_o = \frac{K}{1+K}\,V_i$$

$$V_+ - V_- = V_i - V_o = V_i - \frac{K}{1+K}\,V_i = \frac{1}{1+K}\,V_i = \frac{1}{K}\,V_o$$

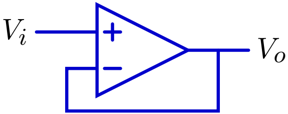
$$\lim_{K\rightarrow\infty}\left(V_+ - V_-\right) = 0$$

If the difference between $V+$ and V_- did not go to zero as $K \rightarrow \infty$ then $V_o = K\ \left(V_+ - V_-\right)$ could not be finite.

The "Ideal" Op-Amp

The approximation that $V_+ = V_-$ is referred to as the "ideal" op-amp approximation. It greatly simplifies analysis.

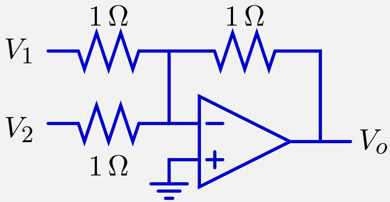
Example.



If $V_+ = V_-$ then $V_o = V_i$!

Check Yourself

Determine the output of the following circuit.

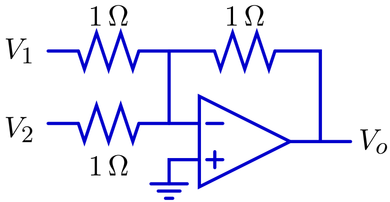


1. $V_o = V_1 + V_2$
2. $V_o = V_1 - V_2$
3. $V_o = -V_1 - V_2$
4. $V_o = -V_1 + V_2$

5 none of the above

Check Yourself

Determine the output of the following circuit.



Ideal op-amp approximation:

$$V_- = V_+ = 0$$

KCL at V_-:

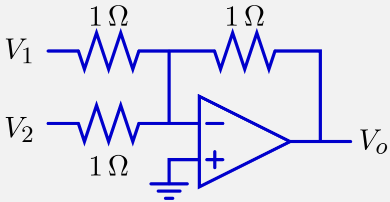
$$\frac{V_1 - 0}{1} + \frac{V_2 - 0}{1} + \frac{V_o - 0}{1} = 0$$

Solving:

$$V_o = -V_1 - V_2$$

Check Yourself

Determine the output of the following circuit. 3

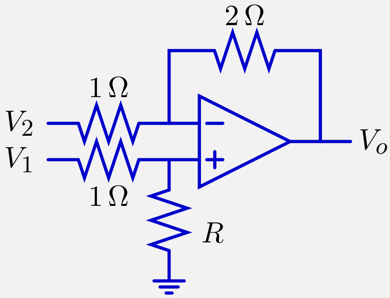


1. $V_o = V_1 + V_2$
2. $V_o = V_1 - V_2$
3. $V_o = -V_1 - V_2$ an inverting summer
4. $V_o = -V_1 + V_2$

5 none of the above

Check Yourself

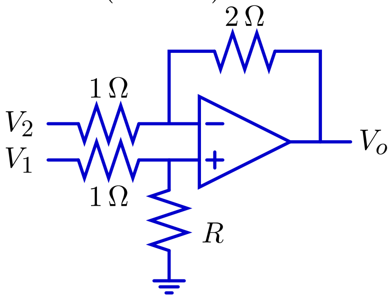
Determine R so that $V_o = 2\left(V_1 - V_2\right)$.



1. $R = 0$
2. $R = 1$
3. $R = 2$
4. $R \to \infty$
5. none of the above

Check Yourself

Determine R so that $V_o = 2\left(V_1 - V_2\right)$.



No current in positive or negative inputs:

$$V_+ = \frac{R}{1+R}\,V_1$$

$$V_- = V_2 + \frac{1}{1+2}\left(V_o - V_2\right) = \frac{2}{3}\,V_2 + \frac{1}{3}\,V_o$$

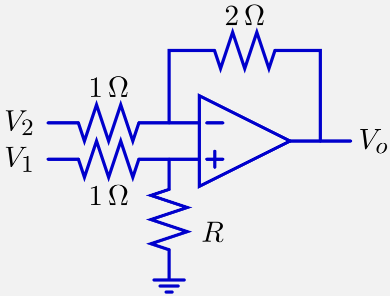
Ideal op-amp:

$$V_+ = V_- = \frac{R}{1+R}\,V_1 = \frac{2}{3}\,V_2 + \frac{1}{3}\,V_o$$

$$V_o = \frac{3R}{1+R}\,V_1 - 2V_2 \qquad \rightarrow \qquad \frac{3R}{1+R} = 2 \qquad \rightarrow \quad R = 2\,\Omega$$

Check Yourself

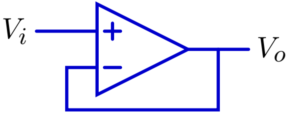
Determine R so that $V_o = 2\left(V_1 - V_2\right)$. 3

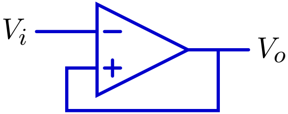


1. $R = 0$
2. $R = 1$
3. $R = 2$
4. $R \to \infty$
5. none of the above

The "Ideal" Op-Amp

The ideal op-amp approximation implies that both of these circuits function identically.





$V_+ = V_- \quad \rightarrow \quad V_o = V_i$!

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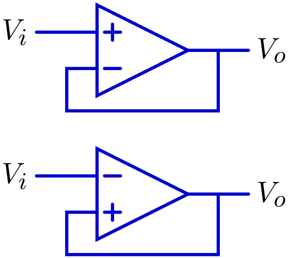
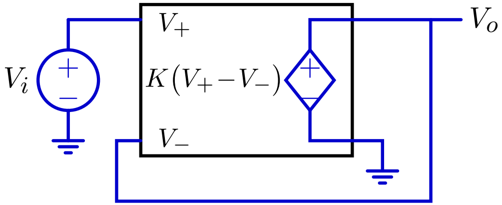


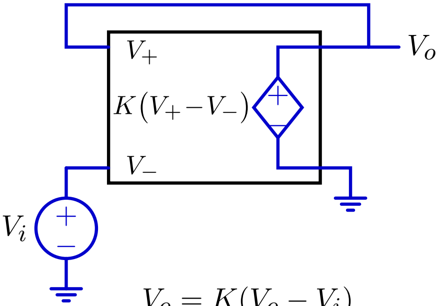
$V_+ = V_- \quad \rightarrow \quad V_o = V_i$!

This sounds a bit implausible!

Paradox

Try analyzing the voltage-controlled voltage source model.





$$V_o = K(V_i - V_o)$$

$$(1+K)V_o = KV_i$$

$$\frac{V_o}{V_i} = \frac{K}{1+K} \approx 1$$

$$V_o = K(V_o - V_i)$$

$$(1-K)V_o = -KV_i$$

$$\frac{V_o}{V_i} = \frac{-K}{1-K} \approx 1$$

These circuits seem to have identical responses if K is large.

Something is wrong!

“Thinking” like an op-amp

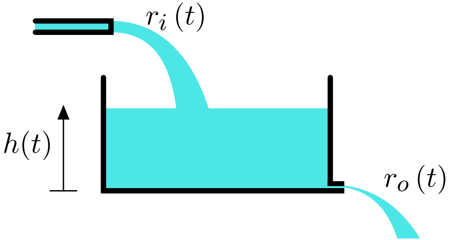
This reasoning is wrong because it ignores a critical property of circuits.

For a voltage to change, charged particles must flow.

To understand flow, we need to understand continuity.

Flows and Continuity

If a quantity is conserved, then the difference between what comes in and what goes out must accumulate.



Flows and Continuity

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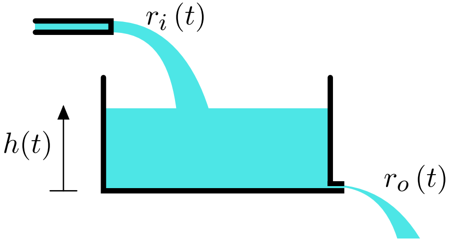
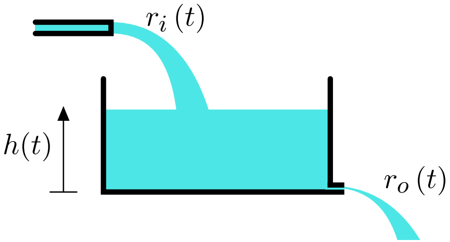


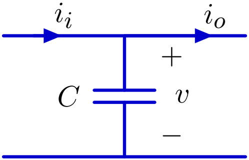
If water is conserved then $\dfrac{dh(t)}{dt} \propto r_i(t) - r_o(t)$.

Leaky Tanks and Capacitors

Water accumulates in a leaky tank.



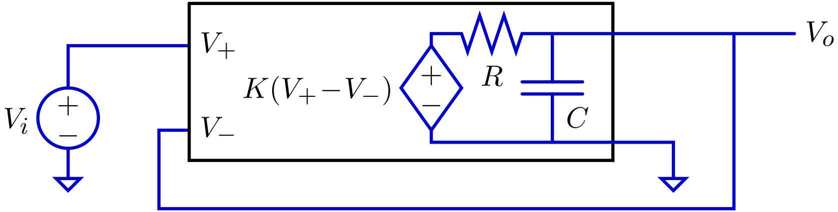
Charge accumulates in a capacitor.



$$\frac{dv}{dt} = \frac{i_i - i_o}{C} \propto i_i - i_o \qquad \text{analogous to} \qquad \frac{dh}{dt} \propto r_i - r_o$$

Charge Accumulation in an Op-Amp

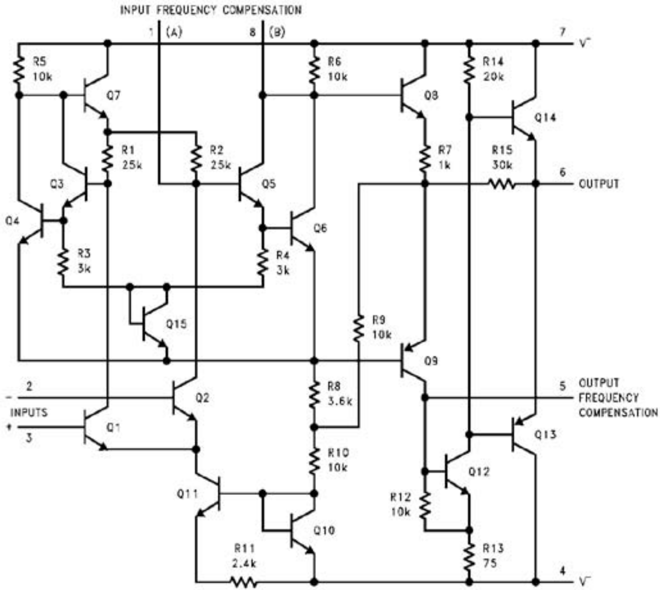
We can add a resistor and capacitor to "model" the accumulation of charge in an op-amp.



This is not an accurate representation of what is inside an op-amp.

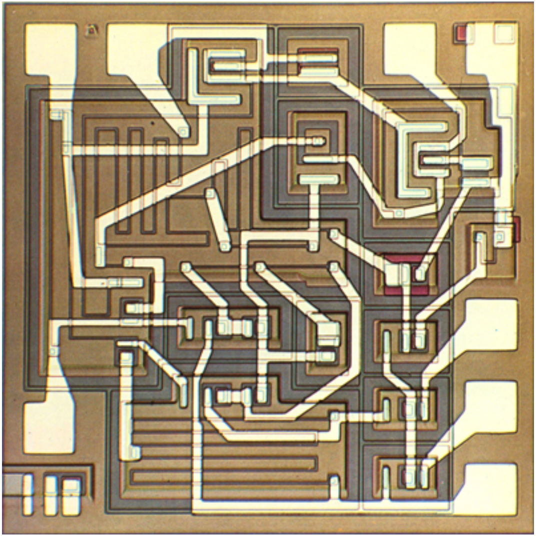
Op-Amp Model

Here is a more accurate circuit model of a μA709 op-amp.



Op-Amp

This artwork shows the physical structure of a μA709 op-amp.



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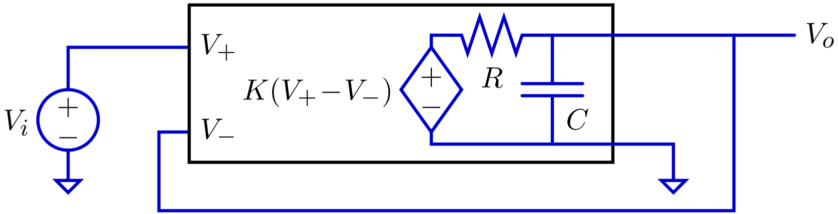


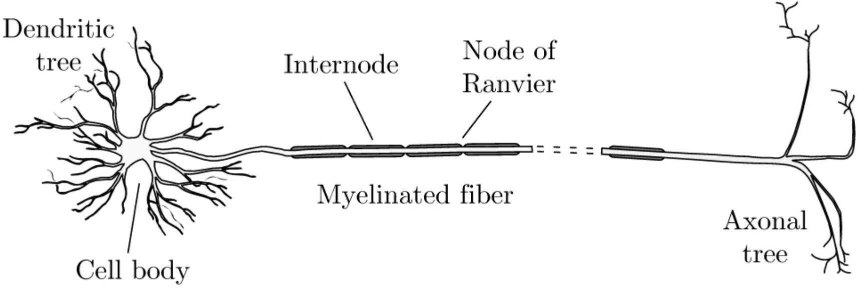
This is not an accurate representation of what is inside an op-amp.

This is a **model** of how the op-amp works.

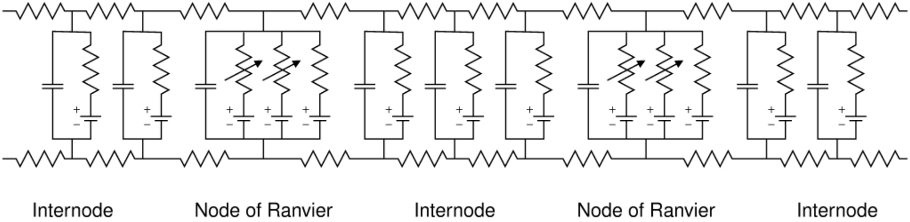
This is an example of using circuits as a tool for modeling.

Circuits as Models

Circuits as models of complex systems: myelinated neuron.

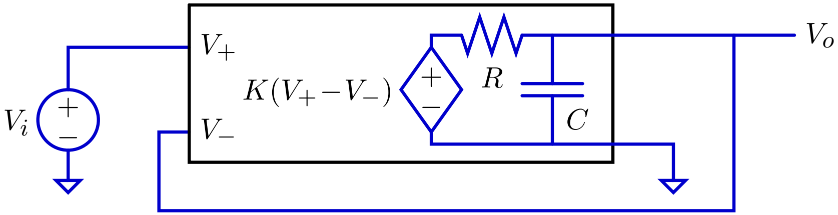


Model of myelinated nerve fiber



Charge Accumulation in an Op-Amp

We can add a resistor and capacitor to "model" the accumulation of charge in an op-amp.



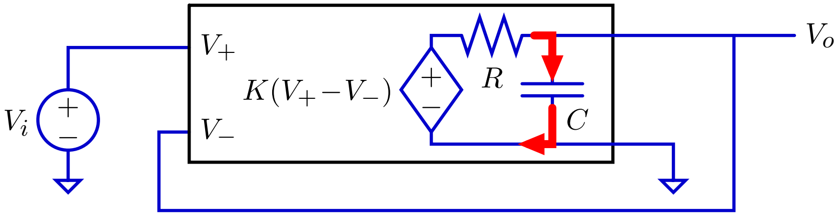
This is not an accurate representation of what is inside an op-amp.

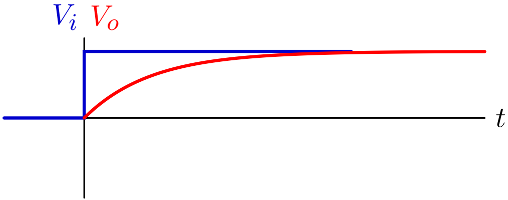
This is a **model** of how the op-amp works.

This is an example of using circuits as a tool for modeling.

Dynamic Analysis of Op-Amp

If the input voltage to this circuit suddenly increases, then current will flow into the capacitor and gradually increase V_o.

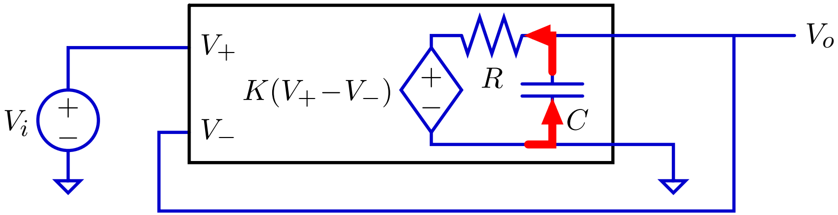


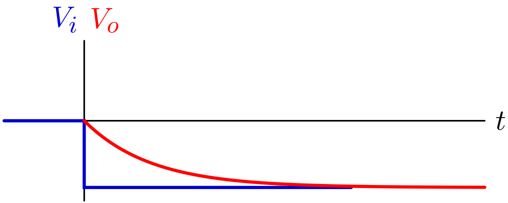


As V_o increases, the difference $V_+ - V_-$ decreases, less current flows, and V_o approaches a final value equal to V_i.

Dynamic Analysis of Op-Amp

If the input voltage to this circuit suddenly decreases, then current will flow out of the capacitor and decrease V_o.

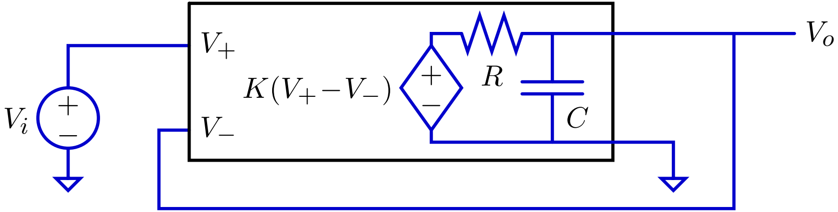


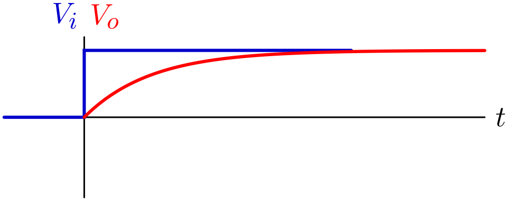


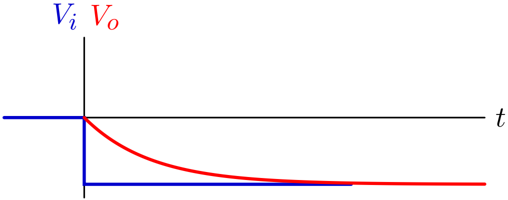
As V_o decreases, the $|V_+ - V_-|$ decreases, the magnitude of the current decreases, and V_o approaches a final value equal to V_i.

Dynamic Analysis of Op-Amp

Regardless of how V_i changes, V_o changes in a direction to reduce the difference between V_i and V_o.

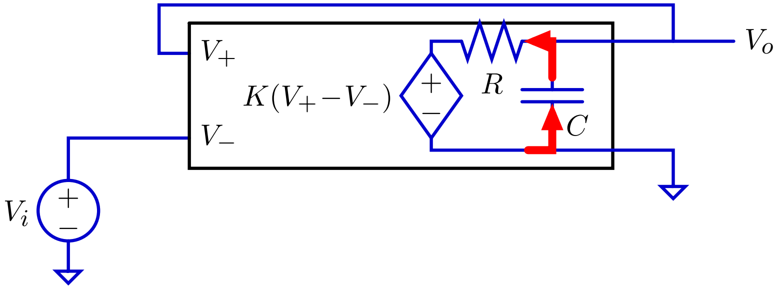


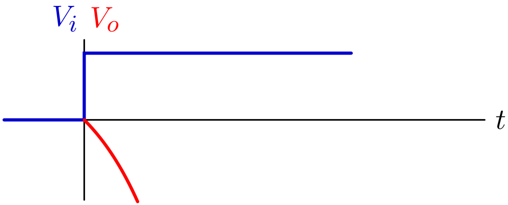




Dynamic Analysis of Op-Amp

Switching the plus and minus inputs flips these relations. Now if the input increases, current will flow out of the capacitor and decrease V_o.

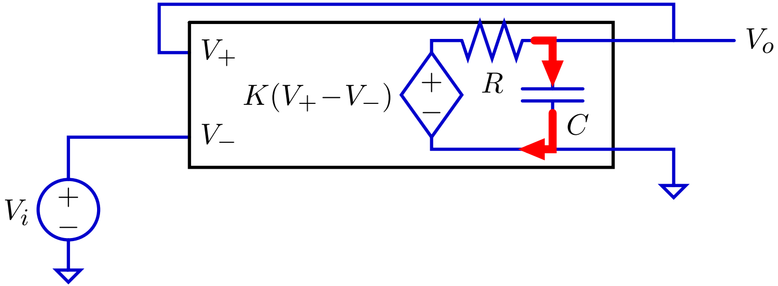


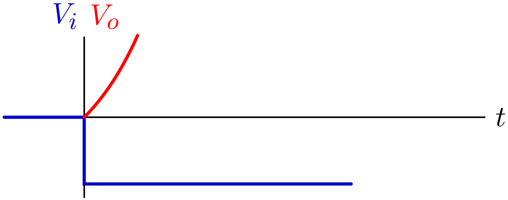


This makes the difference between input and output even bigger!

Dynamic Analysis of Op-Amp

Similarly, if the input decreases, current will flow into the capacitor a increase V_o.



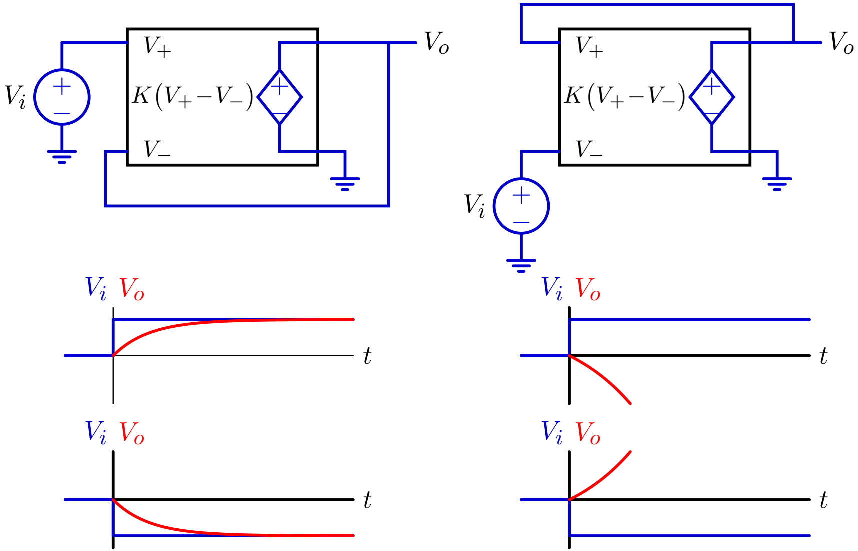


As the output diverges from the input, the magnitude of the capacitor current increases, and the rate of divergence increases!

Positive and Negative Feedback

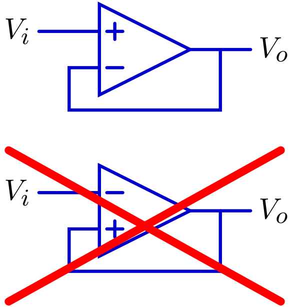
Negative feedback (left) drives the output **toward** the input.

Positive feedback (right) drives the output **away from** the input.



Paradox Resolved

Although both circuits have solutions with $V_o = V_i$ (large K), only the first is stable to changes in V_i.

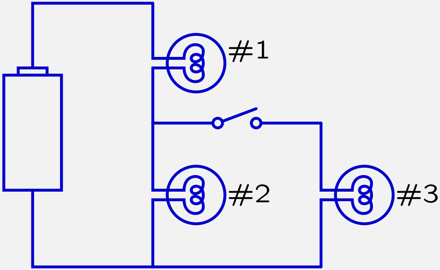


Feedback to the positive input of an op-amp is unstable.

Use negative feedback to get a stable result.

Check Yourself

What happens if we add third light bulb?

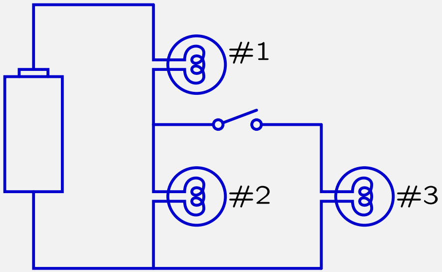


Closing the switch will make

1. bulb 1 brighter
2. bulb 2 dimmer
3. 1 and 2
4. bulbs 1, 2, & 3 equally bright
5. none of the above

Check Yourself

What happens if we add third light bulb? 3

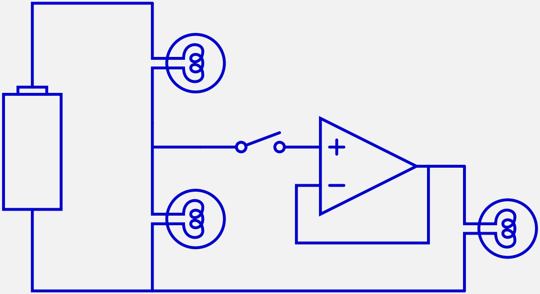


Closing the switch will make

1. bulb 1 brighter
2. bulb 2 dimmer
3. 1 and 2
4. bulbs 1, 2, & 3 equally bright
5. none of the above

Check Yourself

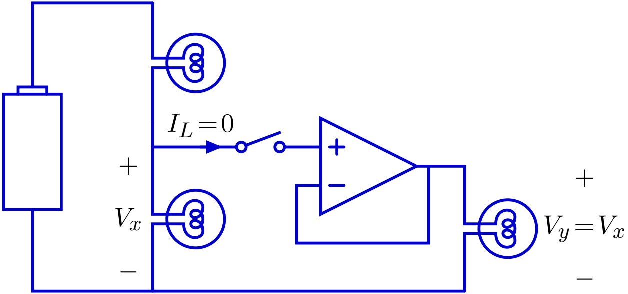
What will happen when the switch is closed?



1. top bulb is brightest
2. right bulb is brightest
1. right bulb is dimmest
4. all 3 bulbs equally bright
5. none of the above

Check Yourself

What will happen when the switch is closed?



Closing the switch will have no effect on the left bulbs because no current will flow ($I_L = 0$) when switch is open OR closed.

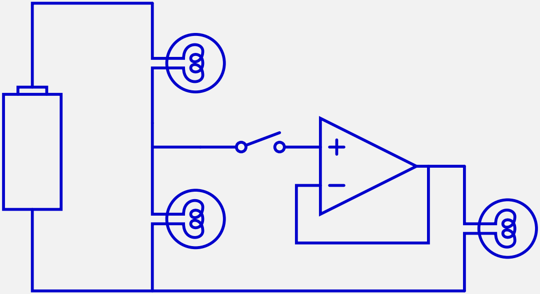
→ This is half of the buffer idea: no input current!

When the switch is closed, $V_y = V_x$.

→ This is the other half: output voltage = input voltage!

Check Yourself

What will happen when the switch is closed? 4

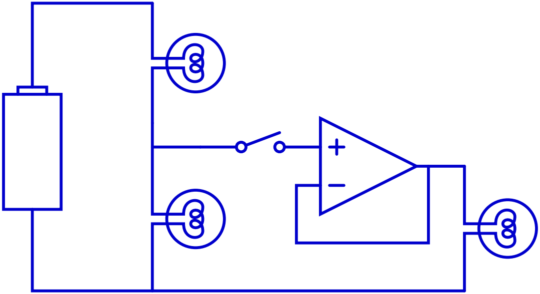


1. top bulb is brightest
2. right bulb is brightest
3. right bulb is dimmest
4. all 3 bulbs equally bright
5. none of the above

Check Yourself

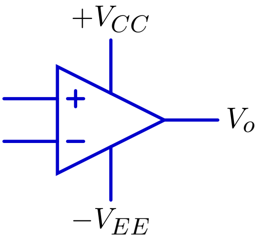
The battery provides the power to illuminate the left bulbs.

Where does the power come from to illuminate the right bulb?



Power Rails

Op-amps derive power from connections to a power supply.



Typically, the output voltage of an op-amp is constrained by the power supply:

$$-V_{EE} < V_o < V_{CC}.$$

Summary

An op-amp can be represented as a voltage-dependent voltage source.

The "ideal" op-amp approximation is $V_+ = V_-$.

The ideal op-amp approximation only makes sense when the op-amp is connected with negative feedback.

6.01SC Introduction to Electrical Engineering and Computer Science
Spring 2011