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6.013/ESD.013J Electromagnetics and Applications, Fall 2005

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Massachusetts Institute of Technology  
 Department of Electrical Engineering and Computer Science  
 6.013 Electromagnetics and Applications  
 Problem Set #1 SOLUTION  
 Fall Term 2005

Problem 1.1

b.  $T = \frac{Mg}{\cos \theta} = \frac{Q_1 Q_2}{4\pi\epsilon_0 s^2 \sin \theta}$  where  $\sin \theta = \frac{s}{2l}$ ,  $\left( Q_1 = Q_2 = \frac{Q}{2} \right)$

$$\frac{Mg 4\pi\epsilon_0 (2l \sin \theta)^2 \sin \theta}{Q_1 Q_2 \cos \theta} = 1, \quad \sin^2 \theta \tan \theta = \frac{Q_1 Q_2}{16\pi\epsilon_0 l^2 Mg} = \frac{Q^2}{64\pi\epsilon_0 l^2 Mg}$$

c.  $i + \frac{d(-q)}{dt} = 0 \Rightarrow i = \frac{dq}{dt}$ ,  $v \approx iR = R \frac{dq}{dt} = -Rq_0 \omega \sin(\omega t)$

Problem 1.2

b. Ampere's integral law  $\oint_{C_b} \vec{H} \cdot d\vec{s} = \int_{S_b} \vec{J} \cdot d\vec{a}$ ,  $H \approx \frac{Ni}{2\pi a}$

c.  $L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{NBS_b}{i} = \frac{\mu_0 N^2 a}{2}$

d.  $v = L \frac{di}{dt}$ ,  $i = C \frac{dv}{dt}$ ,  $v(t=0) = V \Rightarrow i = I \sin(\omega t)$ ,  $\omega = \frac{1}{\sqrt{LC}}$

At  $t=0$ ,  $v(t=0) = V = LI\omega \Rightarrow I = \frac{V}{L\omega} = V \sqrt{\frac{C}{L}}$  ; Note:  $\frac{1}{2} LI^2 = \frac{1}{2} CV^2$

$$I = V \sqrt{\frac{C}{L}} \sin(\omega t), \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

e.  $\frac{1}{2} Mv^2 = \frac{1}{2} CV^2 \Rightarrow v = V \sqrt{\frac{C}{M}}$

$$\frac{1}{2} CV^2 = Mgh \Rightarrow h = \frac{1}{2Mg} CV^2$$

f.  $L = 0.1mH$ ,  $I = 2000A$ ,  $v = 707m/s$ ,  $h = 255m$

Problem 1.3

$$m \frac{d^2 z}{dt^2} = qE_0 \Rightarrow z = \frac{qE_0 t^2}{2m} + v_{z0} t + z_0 = \frac{qE_0 t^2}{2m}, \quad v_{z0} t = z_0 = 0$$

$$m \frac{d^2x}{dt^2} = 0 \Rightarrow x = v_0 t \Rightarrow t = \frac{x}{v_0}$$

$$z = \frac{qE_0 x^2}{2mv_0^2}, z(x=L) = h = \frac{qE_0 L^2}{2mv_0^2}$$

Problem 1.4

- b. The Lorentz force law  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

In the steady state  $\vec{F} = 0$ , so:  $\vec{E} = -\vec{v} \times \vec{B}$

$$\vec{v} = \begin{cases} v_y \vec{i}_y; \text{ positive charge carries} \\ -v_y \vec{i}_y; \text{ negative charge carries} \end{cases}, \vec{B} = B_0 \vec{i}_z$$

$$\vec{E} = \begin{cases} v_y B_0 \vec{i}_x; \text{ positive charge carries} \\ -v_y B_0 \vec{i}_x; \text{ negative charge carries} \end{cases}$$

- c.  $V_H = \Phi(x=d) - \Phi(x=0) = -\int_0^d E_x dx = \int_d^0 E_x dx$

$$V_H = \begin{cases} v_y B_0 d; \text{ positive charge carriers} \\ -v_y B_0 d; \text{ negative charge carriers} \end{cases}$$

- d. As seen in part c, different polarity charge carriers have opposite polarity voltage, so the answer is an indubitable “Yes!”.

Problem 1.5

- a. As the line currents have infinite extent in the z direction the magnetic field has no dependence on the z coordinate.

The magnetic field of a z-directed line current at the origin is:  $\vec{H} = \frac{I}{2\pi r} \vec{i}_\phi$

Convert cylindrical coordinates to Cartesian coordinates and move the line current to  $(0, d/2)$ , the magnetic field is

$$\vec{H} = \frac{I}{2\pi \left( x^2 + \left( y - \frac{d}{2} \right)^2 \right)} \left( -\left( y - \frac{d}{2} \right) \vec{i}_x + x \vec{i}_y \right)$$

Moving the line current to  $(0, -d/2)$  gives the magnetic field as

$$\vec{H} = \frac{I}{2\pi \left( x^2 + \left( y + \frac{d}{2} \right)^2 \right)} \left( -\left( y + \frac{d}{2} \right) \vec{i}_x + x \vec{i}_y \right)$$

The total magnetic field due to the two line currents is

$$\vec{H}_{total} = \frac{I_1}{2\pi \left( x^2 + \left( y - \frac{d}{2} \right)^2 \right)} \left( - \left( y - \frac{d}{2} \right) \vec{i}_x + x \vec{i}_y \right) + \frac{I_2}{2\pi \left( x^2 + \left( y + \frac{d}{2} \right)^2 \right)} \left( - \left( y + \frac{d}{2} \right) \vec{i}_x + x \vec{i}_y \right)$$

b. The force density on a line current (force per length) is  $\vec{F} = \vec{I} \times \vec{B}$ .

At  $(0, d/2)$  the magnetic field is:  $\vec{H} = -\frac{I_2 \vec{i}_x}{2\pi d}$

$$\vec{F} = \vec{I}_1 \times \mu_0 \vec{H}_2 = -\frac{\mu_0 I_1 I_2}{2\pi d} \vec{i}_y$$

c.  $H_x(x, y=0) = \frac{I_1 \frac{d}{2}}{2\pi \left( x^2 + \left( \frac{d}{2} \right)^2 \right)} - \frac{I_2 \frac{d}{2}}{2\pi \left( x^2 + \left( \frac{d}{2} \right)^2 \right)}$ .

When  $I_1 / I_2 = 1$ ,  $H_x(x, y=0) = 0$

$$H_y(x, y=0) = \frac{I_1 x}{2\pi \left( x^2 + \left( \frac{d}{2} \right)^2 \right)} + \frac{I_2 x}{2\pi \left( x^2 + \left( \frac{d}{2} \right)^2 \right)},$$

When  $I_1 / I_2 = -1$ ,  $H_y(x, y=0) = 0$