Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications
Problem Set #1 SOLUTION
Fall Term 2005

Problem 1.1

b. $T = \frac{Mg}{\cos\theta} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 s^2 \sin\theta}$ where $\sin\theta = \frac{s}{2l}$, $\left(Q_1 = Q_2 = \frac{Q}{2}\right)$

$$\frac{Mg 4\pi\varepsilon_0 \left(2l\sin\theta\right)^2 \sin\theta}{Q_1 Q_2 \cos\theta} = 1,\ \sin^2\theta\tan\theta = \frac{Q_1 Q_2}{16\pi\varepsilon_0 l^2 Mg} = \frac{Q^2}{64\pi\varepsilon_0 l^2 Mg}$$

c. $i + \frac{d\left(-q\right)}{dt} = 0 \Rightarrow i = \frac{dq}{dt},\ v \simeq iR = R\frac{dq}{dt} = -Rq_0\omega\sin\left(\omega t\right)$

Problem 1.2

b. Ampere's integral law $\oint_{C_b} \overline{H} \bullet d\overline{s} = \int_{S_b} \overline{J} \bullet d\overline{a}$, $H \approx \frac{Ni}{2\pi a}$

c. $L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{NBS_b}{i} = \frac{\mu_0 N^2 a}{2}$

d. $v = L\frac{di}{dt}, i = C\frac{dv}{dt}, v\left(t=0\right) = V \Rightarrow i = I\sin\left(\omega t\right), \omega = \frac{1}{\sqrt{LC}}$

At $t = 0$, $v\left(t=0\right) = V = LI\omega \Rightarrow I = \frac{V}{L\omega} = V\sqrt{\frac{C}{L}}$; $Note: \frac{1}{2}LI^2 = \frac{1}{2}CV^2$

$$I = V\sqrt{\frac{C}{L}}\sin\left(\omega t\right),\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

e. $\frac{1}{2}Mv^2 = \frac{1}{2}CV^2 \Rightarrow v = V\sqrt{\frac{C}{M}}$

$$\frac{1}{2}CV^2 = Mgh \Rightarrow h = \frac{1}{2Mg}CV^2$$

f. $L = 0.1mH,\ I = 2000A,\ v = 707m/s,\ h = 255m$

Problem 1.3

$$m\frac{d^2 z}{dt^2} = qE_0 \Rightarrow z = \frac{qE_0 t^2}{2m} + v_{z0}t + z_0 = \frac{qE_0 t^2}{2m},\ v_{z0}t = z_0 = 0$$

$$m\frac{d^2x}{dt^2}=0\Rightarrow x=v_0t\Rightarrow t=\frac{x}{v_0}$$

$$z=\frac{qE_0x^2}{2m{v_0}^2},\ z\left(x=L\right)=h=\frac{qE_0L^2}{2m{v_0}^2}$$

Problem 1.4

b. The Lorentz force law $\overline{F}=q\left(\overline{E}+\overline{v}\times\overline{B}\right)$

In the steady state $\overline{F}=0$, so: $\overline{E}=-\overline{v}\times\overline{B}$

$$\overline{v}=\begin{cases}v_y\overline{i}_y;\ \text{postive charge carries}\\-v_y\overline{i}_y;\ \text{negative charge carries}\end{cases},\quad \overline{B}=B_0\overline{i}_z$$

$$\overline{E}=\begin{cases}v_yB_0\overline{i}_x;\ \text{postive charge carries}\\-v_yB_0\overline{i}_x;\ \text{negative charge carries}\end{cases}$$

c. $V_H=\Phi\left(x=d\right)-\Phi\left(x=0\right)=-\int_0^d E_x dx=\int_d^0 E_x dx$

$$V_H=\begin{cases}v_yB_0d;\ \text{positive charge carriers}\\-v_yB_0d;\ \text{negative charge carriers}\end{cases}$$

d. As seen in part c, different polarity charge carriers have opposite polarity voltage, so the answer is an indubitable “Yes!”.

Problem 1.5

a. As the line currents have infinite extent in the z direction the magnetic field has no dependence on the z coordinate.

The magnetic field of a z-directed line current at the origin is: $\overline{H}=\frac{I}{2\pi r}\overline{i}_\phi$

Convert cylindrical coordinates to Cartesian coordinates and move the line current to $\left(0,d/2\right)$, the magnetic field is

$$\overline{H}=\frac{I}{2\pi\left(x^2+\left(y-\frac{d}{2}\right)^2\right)}\left(-\left(y-\frac{d}{2}\right)\overline{i}_x+x\overline{i}_y\right)$$

Moving the line current to $\left(0,-d/2\right)$ gives the magnetic field as

$$\overline{H}=\frac{I}{2\pi\left(x^2+\left(y+\frac{d}{2}\right)^2\right)}\left(-\left(y+\frac{d}{2}\right)\overline{i}_x+x\overline{i}_y\right)$$

The total magnetic field due to the two line currents is

$$\overline{H}_{total} = \frac{I_1}{2\pi\left(x^2+\left(y-\frac{d}{2}\right)^2\right)}\left(-\left(y-\frac{d}{2}\right)\overline{i}_x + x\overline{i}_y\right) + \frac{I_2}{2\pi\left(x^2+\left(y+\frac{d}{2}\right)^2\right)}\left(-\left(y+\frac{d}{2}\right)\overline{i}_x + x\overline{i}_y\right)$$

b. The force density on a line current (force per length) is $\overline{F} = \overline{I} \times \overline{B}$.

At $(0, d/2)$ the magnetic field is: $\overline{H} = -\frac{I_2 \overline{i}_x}{2\pi d}$

$$\overline{F} = \overline{I}_1 \times \mu_0 \overline{H}_2 = -\frac{\mu_0 I_1 I_2}{2\pi d}\overline{i}_y$$

c. $$H_x(x, y=0) = \frac{I_1 \frac{d}{2}}{2\pi\left(x^2+\left(\frac{d}{2}\right)^2\right)} - \frac{I_2 \frac{d}{2}}{2\pi\left(x^2+\left(\frac{d}{2}\right)^2\right)}.$$

When $I_1 / I_2 = 1$, $H_x(x, y=0) = 0$

$$H_y(x, y=0) = \frac{I_1 x}{2\pi\left(x^2+\left(\frac{d}{2}\right)^2\right)} + \frac{I_2 x}{2\pi\left(x^2+\left(\frac{d}{2}\right)^2\right)},$$

When $I_1 / I_2 = -1, H_y(x, y=0) = 0$