Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

Problem Set #11 — Issued: 11/29/2005
Fall Term 2005 — Due: 12/9/05

Suggested Reading Assignment: Staelin, Sections 6.1-6.4, 10.1, 10.2, 10.4

**Final Exam**: Wednesday, Dec. 21, 2005, 1:30-4:30pm.

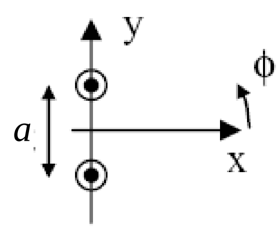
Problem 11.1
A popular 1-MHz AM radio station in the middle of Kansas has a single transmitting antenna on a flat prairie that radiates 100kW isotropically (equally in all directions) over the upper $2\pi$ steradians (i.e., this station has no underground audience.) The matched input impedance (the radiation resistance $R_r$) of this antenna is ~70 ohms, and it is driven by $V_0 \sin \omega t$ volts at maximum power.

a) What is $V_0[\text{Volts}]$?
b) What is the radiated intensity $I[W/m^2]$ 50 kilometers from this antenna?
c) What is the maximum power $P_r$ that can be received from this station by an antenna 50 km away with an effective area $A = 10\ m^2$?

Problem 11.2
A short dipole antenna, 10 cm in length and aligned along the $\hat{z}$ axis, is driven uniformly along its length with a sinusoidal current of peak value 1 amp.

a) What is the electric field $\overline{E}(r,\theta,t)$ in the far field?
b) At what frequency would this antenna radiate 1 watt of power?
c) If a receiver with effective area $A = 0.1\ m^2$ needed $10^{-20}$ watts for successful reception, how far away could it be and still receive signals from the 1 watt dipole? In what direction?

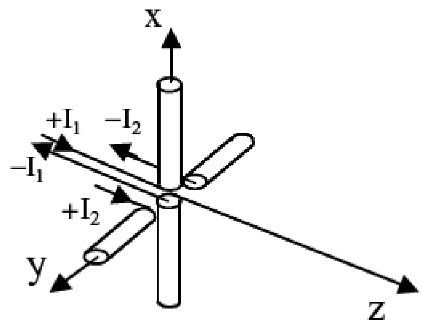
Problem 11.3
An antenna consists of two short dipoles, oriented along the *z*-axis and separated along the *y*-axis by a distance *a*. They are driven in phase, each with a current $I_0$ and an effective length $d_{eff}$, ($d_{eff} \ll \lambda$), at an angular frequency of ω. (Assume that each antenna radiates as it would in the absence of the other.)




a) What is the intensity of the radiation in the far field as a function of angle $\phi$ in the *x*-*y* plane?
b) For $a = 2\lambda$, at what angles $\phi_{max}$ and $\phi_{min}$ is the intensity a relative maximum or zero?

Problem 11.4

A "turnstile" antenna consists of two short Hertzian dipoles driven at an angular frequency ω and oriented at right angles to each other as shown in the figure below. One dipole, oriented along the *x*-axis is driven with a current $\hat{\bar{I}}_1 = \hat{I}_0\hat{x}$ and the other, oriented along the *y*-axis is driven with $\hat{\bar{I}}_2 = j\hat{I}_0\hat{y}$. Both have the same effective length $d_{eff}$.

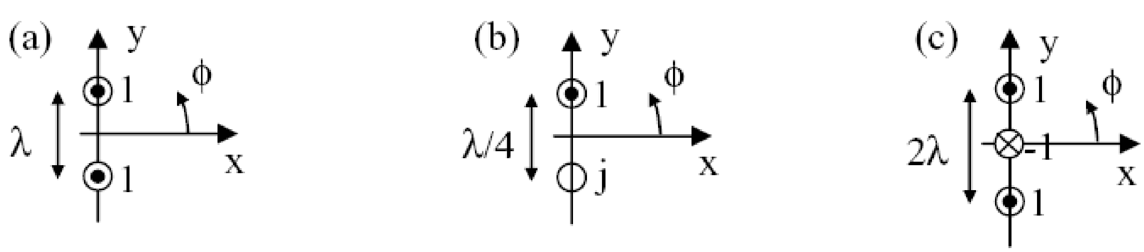



a) Find the complex amplitude of the total electric field on the +*z* axis in the far field. (Express your answer in Cartesian coordinates with unit vectors $\hat{x}, \hat{y},$ and $\hat{z}$.)
b) Why is the result of part (a) called left-handed circular polarization (LHCP) for +*z* directed waves along the +*z* axis?
c) What is the complex amplitude of the magnetic field on the +*z* axis in the far field?
d) What is the intensity of the radiation on the *z* axis in the far field?

   Hint: $\langle \bar{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\hat{\bar{E}} \times \hat{\bar{H}}^*\right]$

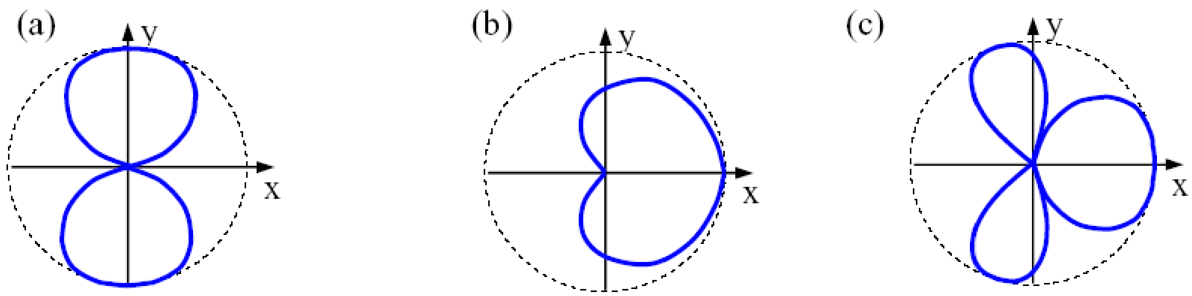
Problem 11.5

Sketch the far field radiation patterns in the *x*-*y* plane for each of the following short dipole antenna arrays. The identical dipoles are directed in either the +*z* ⊙ or -*z* ⊗ directions, as indicated, and the currents have equal amplitudes of $\pm 1$. In part (b) one current has a phase of $\frac{\pi}{2}$ so that its complex amplitude is *j*. In each case find the angles $\phi$ corresponding to nulls ($\phi_n$) and peaks ($\phi_p$). If the peaks are unequal, also evaluate their relative values.



Problem 11.6

Using the format of Problem 11.5 design two-dipole arrays that could produce the far field antenna gain patterns illustrated below. The two dipoles have the same current amplitude but may differ in phase. Find the spacing *a* between the two dipoles and their relative phase that results in the radiation patterns shown in parts (a) - (c).



# 6.013 Final Exam Formula Sheet
# December 21, 2005

## Cartesian Coordinates (x,y,z):

$$\nabla\Psi = \hat{x}\frac{\partial\Psi}{\partial x} + \hat{y}\frac{\partial\Psi}{\partial y} + \hat{z}\frac{\partial\Psi}{\partial z}$$

$$\nabla\bullet\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\overline{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$$

## Cylindrical coordinates (r,φ,z):

$$\nabla\Psi = \hat{r}\frac{\partial\Psi}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial\Psi}{\partial\phi} + \hat{z}\frac{\partial\Psi}{\partial z}$$

$$\nabla\bullet\overline{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\overline{A} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial\phi}\right) = \frac{1}{r}\det\begin{vmatrix}\hat{r} & r\hat{\phi} & \hat{z}\\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z\\ A_r & rA_\phi & A_z\end{vmatrix}$$

$$\nabla^2\Psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial^2\Psi}{\partial z^2}$$

## Spherical coordinates (r,θ,φ):

$$\nabla\Psi = \hat{r}\frac{\partial\Psi}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial\Psi}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\phi}$$

$$\nabla\bullet\overline{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\overline{A} = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right) + \hat{\theta}\left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)$$

$$= \frac{1}{r^2\sin\theta}\det\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi}\\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\\ A_r & rA_\theta & r\sin\theta A_\phi\end{vmatrix}$$

$$\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}$$

| **Gauss' Divergence Theorem:** | **Vector Algebra:** |
|---|---|
| $\int_V \nabla\bullet\overline{G}\,dv = \oint_A \overline{G}\bullet\hat{n}\,da$ | $\nabla = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$<br>$\overline{A}\bullet\overline{B} = A_xB_x + A_yB_y + A_zB_z$ |
| **Stokes' Theorem:** | $\nabla\bullet(\nabla\times\overline{A}) = 0$ |
| $\int_A (\nabla\times\overline{G})\bullet\hat{n}\,da = \oint_C \overline{G}\bullet d\overline{\ell}$ | $\nabla\times(\nabla\times\overline{A}) = \nabla(\nabla\bullet\overline{A}) - \nabla^2\overline{A}$ |

# Basic Equations for Electromagnetics and Applications

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|---|---|
| **Fundamentals** | |
| $\overline{f} = q\left(\overline{E} + \overline{v} \times \mu_o \overline{H}\right)[N]$ (Force on point charge) | $\overline{E}_{1//} - \overline{E}_{2//} = 0$ |
| $\nabla \times \overline{E} = -\partial \overline{B}/\partial t$ | $\overline{H}_{1//} - \overline{H}_{2//} = \overline{J}_s \times \hat{n}$ |
| $\oint_c \overline{E} \bullet d\overline{s} = -\frac{d}{dt}\int_A \overline{B} \bullet d\overline{a}$ | $B_{1\perp} - B_{2\perp} = 0$   |
| $\nabla \times \overline{H} = \overline{J} + \partial \overline{D}/\partial t$ | $\hat{n} \bullet \left(D_{1\perp} - D_{2\perp}\right) = \rho_s$ |
| $\oint_c \overline{H} \bullet d\overline{s} = \int_A \overline{J} \bullet d\overline{a} + \frac{d}{dt}\int_A \overline{D} \bullet d\overline{a}$ | ↳ 0 = if $\sigma = \infty$ |
| $\nabla \bullet \overline{D} = \rho \rightarrow \int_A \overline{D} \bullet d\overline{a} = \int_V \rho dv$ | **Electromagnetic Quasistatics** |
| $\nabla \bullet \overline{B} = 0 \rightarrow \int_A \overline{B} \bullet d\overline{a} = 0$ | $\overline{E} = -\nabla\Phi(r), \Phi(r) = \int_{V'}\left(\rho(\overline{r})/4\pi\varepsilon \mid \overline{r}' - \overline{r} \mid\right)dv'$ |
| $\nabla \bullet \overline{J} = -\partial\rho/\partial t$ | $\nabla^2\Phi = \frac{-\rho_f}{\varepsilon}$ |
| $\overline{E}$ = electric field ($Vm^{-1}$) | $C = Q/V = A\varepsilon/d$ [F] |
| $\overline{H}$ = magnetic field ($Am^{-1}$) | $L = \Lambda/I$ |
| $\overline{D}$ = electric displacement ($Cm^{-2}$) | $i(t) = C\, dv(t)/dt$ |
| $\overline{B}$ = magnetic flux density (T) | $v(t) = L\, di(t)/dt = d\Lambda/dt$ |
| Tesla (T) = Weber $m^{-2}$ = 10,000 gauss | $w_e = Cv^2(t)/2$; $w_m = Li^2(t)/2$ |
| $\rho$ = charge density ($Cm^{-3}$) | $L_{solenoid} = N^2\mu A/W$ |
| $\overline{J}$ = current density ($Am^{-2}$) | $\tau = RC$, $\tau = L/R$ |
| $\sigma$ = conductivity (Siemens $m^{-1}$) | $\Lambda = \int_A \overline{B} \bullet d\overline{a}$ (per turn) |
| $\overline{J}_s$ = surface current density ($Am^{-1}$) | $KCL: \sum_i I_i(t) = 0$ at node |
| $\rho_s$ = surface charge density ($Cm^{-2}$) | $KVL: \sum_i V_i(t) = 0$ around loop |
| $\varepsilon_o = 8.85 \times 10^{-12}\ Fm^{-1}$ | $Q = \omega_0 w_T / P_{diss} = \omega_0 / \Delta\omega$ |
| $\mu_o = 4\pi \times 10^{-7}\ Hm^{-1}$ | $\omega_0 = (LC)^{-0.5}$ |
| $c = (\varepsilon_o\mu_o)^{-0.5} \cong 3 \times 10^8\ ms^{-1}$ | $\left\langle V^2(t)\right\rangle / R = kT$ |
| $e = -1.60 \times 10^{-19}$ C | |
| $\eta_o \cong 377$ ohms $= (\mu_o/\varepsilon_o)^{0.5}$ | **Electromagnetic Waves** |
| $\left(\nabla^2 - \mu\varepsilon\partial^2/\partial t^2\right)\overline{E} = 0$ [Wave Eqn.] | $\left(\nabla^2 - \mu\varepsilon\partial^2/\partial t^2\right)\overline{E} = 0$ [Wave Eqn.] |
| $E_y(z,t) = E_+(z-ct) + E_-(z+ct) = R_e\{\underline{E}_y(z)e^{j\omega t}\}$ | $\left(\nabla^2 + k^2\right)\hat{\overline{E}} = 0,\ \hat{\overline{E}} = \hat{\overline{E}}_o e^{-j\overline{k}\bullet\overline{r}}$ |
| $H_x(z,t) = \eta_o^{-1}[E_+(z-ct) - E_-(z+ct)]$ [or($\omega t - kz$) or ($t - z/c$)] | $k = \omega(\mu\varepsilon)^{0.5} = \omega/c = 2\pi/\lambda$ |
| $\int_A\left(\overline{E} \times \overline{H}\right) \bullet d\overline{a} + (d/dt)\int_V\left(\varepsilon\lvert\overline{E}\rvert^2/2 + \mu\lvert\overline{H}\rvert^2/2\right)dv$ | $k_x^2 + k_y^2 + k_z^2 = k_o^2 = \omega^2\mu\varepsilon$ |
| $= -\int_V \overline{E} \bullet \overline{J}\ dv$ (Poynting Theorem) | $v_p = \omega/k$, $v_g = (\partial k/\partial\omega)^{-1}$ |
| | $\theta_r = \theta_i$ |
| **Media and Boundaries** | $\sin\theta_t / \sin\theta_i = k_i / k_t = n_i / n_t$ |
| $\overline{D} = \varepsilon_o\overline{E} + \overline{P}$ | $\theta_c = \sin^{-1}\left(n_t / n_i\right)$ |
| $\nabla \bullet \overline{D} = \rho_f$, $\tau = \varepsilon/\sigma$ | $\theta_B = \tan^{-1}\left(\varepsilon_t / \varepsilon_i\right)^{0.5}$ for TM |
| $\nabla \bullet \varepsilon_o\overline{E} = \rho_f + \rho_p$ | $\theta > \theta_c \Rightarrow \hat{\overline{E}}_t = \hat{\overline{E}}_i T e^{+\alpha x - jk_z z}$ |
| $\nabla \bullet \overline{P} = -\rho_p$, $\overline{J} = \sigma\overline{E}$ | $\overline{k} = \overline{k}' - j\overline{k}''$ |
| $\overline{B} = \mu\overline{H} = \mu_o\left(\overline{H} + \overline{M}\right)$ | $\Gamma = T - 1$ |
| $\varepsilon(\omega) = \varepsilon\left(1 - \omega_p^2/\omega^2\right)$, $\omega_p = \left(Ne^2/m\varepsilon\right)^{0.5}$ (**plasma**) | $T_{TE} = 2/\left(1 + \left[\eta_i\cos\theta_t / \eta_t\cos\theta_i\right]\right)$ |
| $\varepsilon_{eff} = \varepsilon\left(1 - j\sigma/\omega\varepsilon\right)$ | $T_{TM} = 2/\left(1 + \left[\eta_t\cos\theta_t / \eta_i\cos\theta_i\right]\right)$ |

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| Skin depth δ = $(2/\omega\mu\sigma)^{0.5}\,[m]$ | |
| Radiating Waves | Wireless Communications and Radar |
| $\nabla^2\overline{A} - \frac{1}{c^2}\frac{\partial^2\overline{A}}{\partial t^2} = -\mu J_f$ | $G(\theta,\phi) = P_r/(P_R/4\pi r^2)$ |
| $\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho_f}{\varepsilon}$ | $P_R = \int_{4\pi} P_r(\theta,\phi,r)\, r^2 \sin\theta\, d\theta d\phi$ |
| $\overline{A} = \int_{V'} \frac{\mu J_f (t - r_{QP}/c) dV'}{4\pi r_{QP}}$ | $P_{rec} = P_r(\theta,\phi)A_e(\theta,\phi)$ |
| $\Phi = \int_{V'} \frac{\rho_f (t - r_{QP}/c) dV'}{4\pi\varepsilon r_{QP}}$ | $A_e(\theta,\phi) = G(\theta,\phi)\lambda^2/4\pi$ |
| $\overline{E} = -\nabla\Phi - \frac{\partial\overline{A}}{\partial t},\ \overline{B} = \nabla\times\overline{A}$ | $G(\theta,\phi) = 1.5\sin^2\theta$ (Hertzian Dipole) |
| $\hat{\Phi}(r) = \int_{V'} \hat{\rho}(\overline{r}) e^{-jk\lvert\overline{r}'-\overline{r}\rvert} / \left(4\pi\varepsilon\lvert\overline{r}'-\overline{r}\rvert\right) dV'$ | $R_r = P_R / \langle i^2(t)\rangle$ |
| $\hat{\overline{A}}(r) = \int_{V'} \left(\mu\hat{\overline{J}}(\overline{r}) e^{-jk\lvert\overline{r}'-\overline{r}\rvert} / 4\pi\lvert\overline{r}'-\overline{r}\rvert\right) dV'$ | $E_{ff}(\theta \cong 0) = \left(je^{jkr}/\lambda r\right)\int_A E_t(x,y) e^{jk_x x + jk_y y} dxdy$ |
| $\hat{E}_{ff\theta} = \sqrt{\frac{\mu}{\varepsilon}}\hat{H}_{ff\phi} = \left(j\eta k\hat{I}d/4\pi r\right)e^{-jkr}\sin\theta$ | $\hat{\overline{E}}_Z = \sum_i a_i \overline{E} e^{-jkr_i}$ = (element factor)(array f) |
| $\nabla^2\hat{\Phi} + \omega^2\mu\varepsilon\hat{\Phi} = -\hat{\rho}/\varepsilon,\ \ \Phi(x,y,z,t) = \mathrm{Re}\left[\hat{\Phi}(x,y,z)e^{j\omega t}\right]$ | $E_{bit} \geq \sim 4 \times 10^{-20}$ [J] |
| $\nabla^2\hat{\overline{A}} + \omega^2\mu\varepsilon\hat{\overline{A}} = -\mu\hat{\overline{J}},\ \ \overline{A}(x,y,z,t) = \mathrm{Re}\left[\hat{\overline{A}}(x,y,z)e^{j\omega t}\right]$ | $\underline{Z}_{12} = \underline{Z}_{21}$ if reciprocity |
| | At $\omega_o$, $\langle w_e\rangle = \langle w_m\rangle$ |
| Forces, Motors, and Generators | $\langle w_e\rangle = \int_V \left(\varepsilon\lvert\hat{\overline{E}}\rvert^2/4\right) dv$ |
| $\overline{J} = \sigma(\overline{E} + \overline{v}\times\overline{B})$ | $\langle w_m\rangle = \int_V \left(\mu\lvert\hat{\overline{H}}\rvert^2/4\right) dv$ |
| $\overline{F} = \overline{I}\times\overline{B}\ [Nm^{-1}]$ (force per unit length) | $Q_n = \omega_n w_{Tn}/P_n = \omega_n/2\alpha_n$ |
| $\overline{E} = -\overline{v}\times\overline{B}$ inside perfectly conducting wire $(\sigma\to\infty)$ | $f_{mnp} = (c/2)\left([m/a]^2 + [n/b]^2 + [p/d]^2\right)^{0.5}$ |
| Max $f/A = B^2/2\mu,\ D^2/2\varepsilon\ [Nm^{-2}]$ | $s_n = j\omega_n - \alpha_n$ |
| $vi = \frac{dw_T}{dt} + f\frac{dz}{dt}$ | |
| $f = ma = d(mv)/dt$ | Acoustics |
| $P = fv = T\omega$ (Watts) | $P = P_o + p,\ \overline{U} = \overline{U}_o + u$ |
| $T = I\, d\omega/dt$ | $\nabla p = -\rho_o \partial\overline{u}/\partial t$ |
| $I = \sum_i m_i r_i^2$ | $\nabla\bullet\overline{u} = -(1/\gamma P_o)\partial p/\partial t$ |
| $\overline{F}_E = \lambda\overline{E}\left[Nm^{-1}\right]$ Force per unit length on line charge λ | $\left(\nabla^2 - k^2\partial^2/\partial t^2\right)p = 0$ |
| $W_M(\lambda,x) = \frac{1}{2}\frac{\lambda^2}{L(x)};\ W_E(q,x) = \frac{1}{2}\frac{q^2}{C(x)}$ | $k^2 = \omega^2/c_s^2 = \omega^2\rho_o/\gamma P_o$ |
| $f_M(\lambda,x) = -\frac{\partial W_M}{\partial x}\Big\rvert_\lambda = -\frac{1}{2}\lambda^2\frac{d}{dx}\left(1/L(x)\right) = \frac{1}{2}I^2\frac{dL(x)}{dx}$ | $c_s = v_p = v_g = (\gamma P_o/\rho_o)^{0.5}$ or $(K/\rho_o)^{0.5}$ |
| $f_E(q,x) = -\frac{\partial W_E}{\partial x}\Big\rvert_q = -\frac{1}{2}q^2\frac{d}{dx}\left(1/C(x)\right) = \frac{1}{2}v^2\frac{dC(x)}{dx}$ | $\eta_s = p/u = \rho_o c_s = (\rho_o\gamma P_o)^{0.5}$ gases |
| | $\eta_s = (\rho_o K)^{0.5}$ solids, liquids |
| Optical Communications | $p, \overline{u}_\perp$ continuous at boundaries |

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| E = hf, photons or phonons | $\underline{p} = \underline{p}_+ e^{-jkz} + \underline{p}_- e^{+jkz}$ |
| hf/c = momentum [kg ms$^{-1}$] | $\underline{u}_z = \eta_s^{-1}(\underline{p}_+ e^{-jkz} - \underline{p}_- e^{+jkz})$ |
| $dn_2/dt = -\left[An_2 + B(n_2 - n_1)\right]$ | $\int_A \overline{u}p \bullet d\overline{a} + (d/dt)\int_V \left(\rho_o \lvert\overline{u}\rvert^2/2 + p^2/2\gamma P_o\right)dV$ |
| | |
| **Transmission Lines** | |
| Time Domain | |
| $\partial v(z,t)/\partial z = -L\partial i(z,t)/\partial t$ | |
| $\partial i(z,t)/\partial z = -C\partial v(z,t)/\partial t$ | |
| $\partial^2 v/\partial z^2 = LC\, \partial^2 v/\partial t^2$ | |
| $v(z,t) = V_+(t - z/c) + V_-(t + z/c)$ | |
| $i(z,t) = Y_o[V_+(t - z/c) - V_-(t + z/c)]$ | |
| $c = (LC)^{-0.5} = (\mu\varepsilon)^{-0.5}$ | |
| $Z_o = Y_o^{-1} = (L/C)^{0.5}$ | |
| $\Gamma_L = V_-/V_+ = (R_L - Z_o)/(R_L + Z_o)$ | |
| Frequency Domain | |
| $(d^2/dz^2 + \omega^2 LC)\hat{V}(z) = 0$ | |
| $\hat{V}(z) = \hat{V}_+ e^{-jkz} + \hat{V}_- e^{+jkz}$, $v(z,t) = \mathrm{Re}\left[\hat{V}(z)e^{j\omega t}\right]$ | |
| $\hat{I}(z) = Y_0[\hat{V}_+ e^{-jkz} - \hat{V}_- e^{+jkz}]$, $i(z,t) = \mathrm{Re}\left[\hat{I}(z)e^{j\omega t}\right]$ | |
| $k = 2\pi/\lambda = \omega/c = \omega(\mu\varepsilon)^{0.5}$ | |
| $Z(z) = \hat{V}(z)/\hat{I}(z) = Z_o Z_n(z)$ | |
| $Z_n(z) = \left[1+\Gamma(z)\right]/\left[1-\Gamma(z)\right] = R_n + jX_n$ | |
| $\Gamma(z) = (V_-/V_+)e^{2jkz} = \left[Z_n(z)-1\right]/\left[Z_n(z)+1\right]$ | |
| $Z(z) = Z_o\left(Z_L - jZ_o \tan kz\right)/\left(Z_o - jZ_L \tan kz\right)$ | |
| $VSWR = \lvert V_{max}\rvert/\lvert V_{min}\rvert$ | |
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