6.013/ESD.013J — Electromagnetics and Applications Fall 2005

Problem Set 10 - Solutions

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Problem 10.1

A

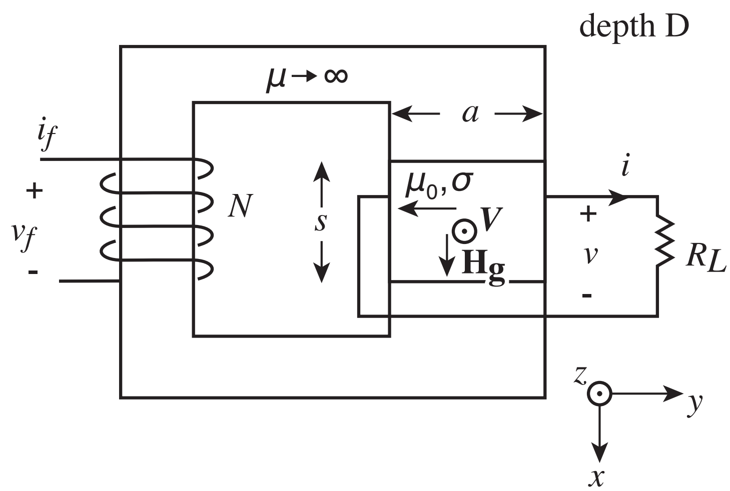

Figure 1: MHD machine in magnetic circuit. (Image by MIT OpenCourseWare.)

$$\mathbf{H}_g = \frac{NI_0}{s}\hat{\mathbf{x}}$$

$$\mathbf{E}_g = -\mathbf{V} \times \mu_0 \frac{NI_0}{s}\hat{\mathbf{x}} = -V_0\hat{\mathbf{z}} \times \mu_0 \frac{NI_0}{s}\hat{\mathbf{x}} = -\hat{\mathbf{y}}\frac{\mu_0 NI_0 V_0}{s}$$

Open circuit voltage:

$$v_{oc} = \frac{\mu_0 NI_0}{s}aV_0$$

$$\lambda = I_0 L_f = \mu_0 N H_x aD = \frac{\mu_0 N^2 I_0}{s}aD \implies L_f = \frac{\mu_0 N^2 aD}{s}$$

Internal resistance:

$$R_i = \frac{a}{sD\sigma}$$

Model:

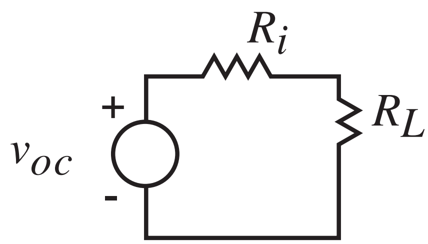

Figure 2: Resistance model. (Image by MIT OpenCourseWare.)

$$P_L = \left(\frac{v_{oc}}{R_i + R_L}\right)\frac{v_{oc}}{(R_i + R_L)}R_L = \frac{R_L v_{oc}^2}{(R_i + R_L)^2} = \frac{R_L}{(R_L + \frac{a}{sD\sigma})^2}\left(\frac{\mu_0 N I_0 a V_0}{s}\right)^2$$

B

Model:

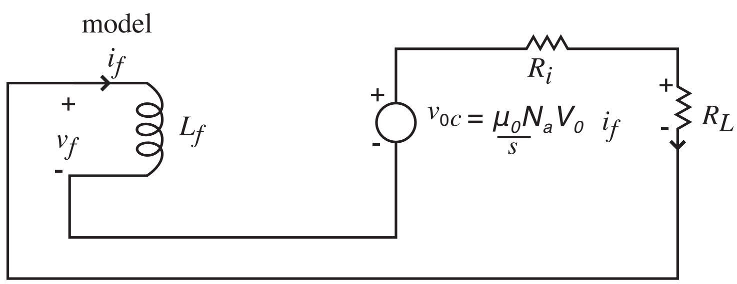

Figure 3: Circuit model. (Image by MIT OpenCourseWare.)

$$L_f \frac{di_f}{dt} + i_f(R_L + R_i) - v_{oc} = 0$$

$$L_f \frac{di_f}{dt} + i_f\left(R_L + R_i - \frac{\mu_0 N a V_0}{s}\right) = 0$$

Unstable if

$$R_L + R_i - \frac{\mu_0 N a V_0}{s} < 0 \implies V_0 = \frac{s}{\mu_0 N a}(R_L + R_i)$$

Problem 10.2

$$(L_r + L_f)\frac{di_f}{dt} + i_f(R_r + R_f - G\omega) + \frac{1}{C}\int i_f \; dt = 0$$

$$\frac{d^2 i_f}{dt^2} + \frac{[R_r + R_f - G\omega]}{(L_r + L_f)}\frac{di_f}{dt} + \frac{1}{(L_f + L_r)C}i_f = 0$$

$$s^2 + \frac{[R_r + R_f - G\omega]}{(L_r + L_f)} s + \frac{1}{(L_r + L_f)C} = 0$$

$$s = -\frac{[R_r + R_f - G\omega]}{2(L_f + L_r)} \pm \sqrt{\left(\frac{[R_r + R_f - G\omega]}{2(L_f + L_r)}\right)^2 - \frac{1}{(L_r + L_f)C}}$$

A

Self excites if $G\omega > R_f + R_r \implies \omega > (R_f + R_r)/G$.

B

$$C > \frac{4(L_f + L_r)}{(R_f + R_r - G\omega)^2} \implies \text{dc self excited}$$

$$C < \frac{4(L_f + L_r)}{(R_f + R_r - G\omega)^2} \implies \text{ac self excited}$$

C

$$\omega_0 = \sqrt{\frac{1}{(L_r + L_f)C} - \left(\frac{R_r + R_f - G\omega}{2(L_r + L_f)}\right)^2}$$

Problem 10.3

A

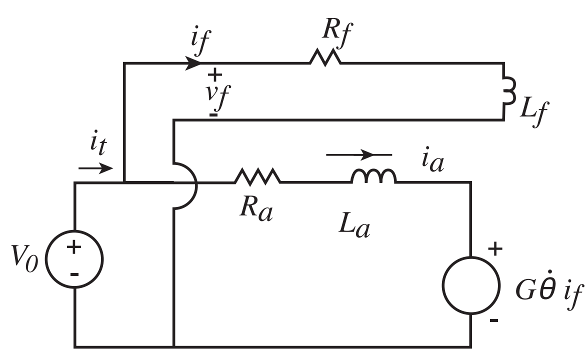

Figure 4: Commutator machine equivalent circuit. (Image by MIT OpenCourseWare.)

Shunt
DC steady state - No voltage drop across inductance

$$i_f = \frac{V_0}{R_f},\ i_a = \frac{V_0 - G\dot{\theta} i_f}{R_a} = \frac{V_0\left(1 - \frac{G\dot{\theta}}{R_f}\right)}{R_a}$$

$$i_t = i_f + i_a = V_0\left(\frac{1}{R_f} + \frac{1}{R_a}\left(1 - \frac{G\dot{\theta}}{R_f}\right)\right)$$

$$T^l = G i_f i_a = \frac{G V_0^2}{R_f R_a}\left(1 - \frac{G\dot{\theta}}{R_f}\right)$$

Series

$$i_f = i_a = i_t = \frac{V_0 - G\dot{\theta} i_f}{R_f + R_a}$$

$$i_t = i_f = \frac{V_0}{R_f + R_a + G\dot{\theta}}$$

$$T_e = \frac{G V_0^2}{(R_f + R_a + G\dot{\theta})^2}$$

B

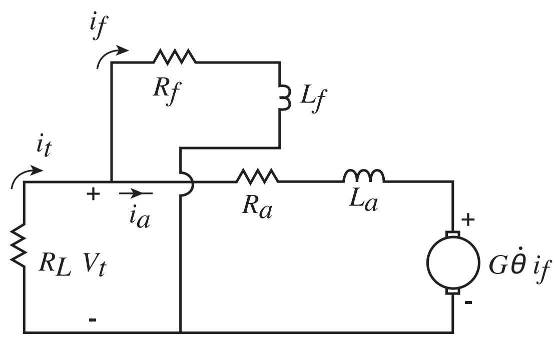

Figure 5: Circuit configuration with shunt load resistor. (Image by MIT OpenCourseWare.)

Shunt

$$i_t = i_f + i_a$$

$$(i_f + i_a)R_L + G\dot{\theta} i_f + L_a \frac{di_a}{dt} + R_a i_a = 0$$

$$(i_f + i_a)R_L + L_f \frac{di_f}{dt} + R_f i_f = 0$$

$$(R_L + R_a)I_a + [R_L + G\dot{\theta}]I_f + sL_a I_a = 0$$

$$R_L I_a + (R_L + sL_f + R_f)I_f = 0$$

$$\frac{I_a}{I_f} = -\frac{(R_L + R_f + sL_f)}{R_L} = -\frac{(R_L + G\dot{\theta})}{(R_L + R_a + L_a s)}$$

$$\implies (R_L + R_f + L_f s)(R_L + R_a + L_a s) = R_L(R_L + G\dot{\theta})$$

$$\implies s^2 + s\frac{[L_f(R_L + R_a) + L_a(R_L + R_f)]}{L_a L_f} + \frac{R_L(R_a + R_f) + R_f R_a - R_L G\dot{\theta}}{L_a L_f} = 0$$

$$\iff s^2 + bs + c = 0$$

$$b = \frac{L_a(R_L + R_f) + L_f(R_L + R_a)}{L_a L_f}$$

$$c = \frac{R_L(R_a + R_f) + R_f R_a - R_L G\dot{\theta}}{L_a L_f}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Unstable if $c < 0$

$$G\dot{\theta} > \frac{R_L(R_a + R_f) + R_f R_a}{R_L}$$

Series

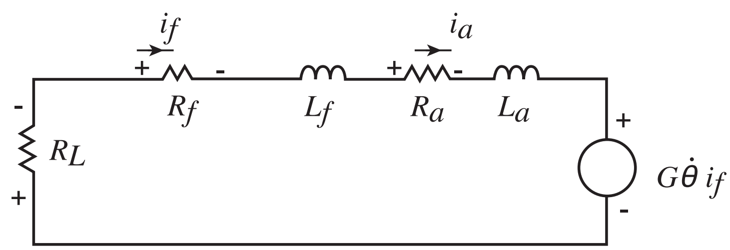

Figure 6: Circuit configuration with load resistor in series. (Image by MIT OpenCourseWare.)

$$(L_f + L_a)\frac{di_f}{dt} + (R_f + R_a)i_f + R_L i_f + G\dot{\theta} i_f = 0$$

$$s(L_f + L_a) + (R_f + R_a + R_L + G\dot{\theta}) = 0$$

$$\implies s = -\frac{R_f + R_a + R_L + G\dot{\theta}}{L_f + L_a}$$

$$\dot{\theta} < -\left(\frac{R_f + R_a + R_L}{G}\right) \text{ for self excited}$$