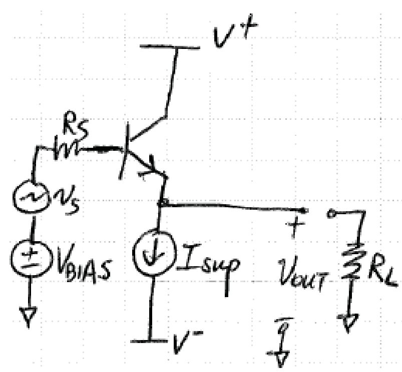
Recitation 23: Frequency Response of Common Collector & Common-Base Amplifier

Yesterday, we used OCT technique for the frequency response of Common-Drain and Common-Gate amplifiers. Today we will look at C-C, C-B frequency response.

Common-Collector Amplifier



One way to study the frequency response is to

- First find the small signal equivalent model for the circuit
- Do KCL, KVL nodal analysis, to find $CO_{3\text{dB}}$
- Or use OCT + Miller Approximation to find $w_{3\text{dB}}$

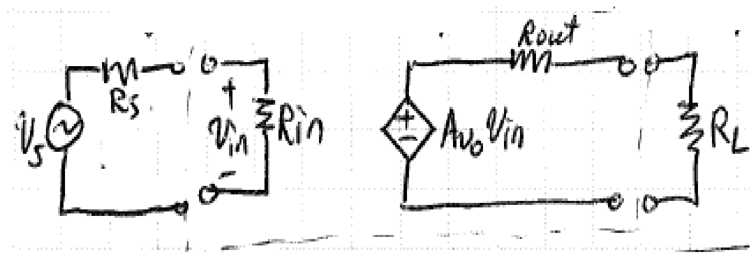
However, the small signal model of this circuit is quite complicated (as the C-D Amp. we talked about yesterday). What we can do is directly use the two-port model for the circuit, and add in the capacitances. So the methodology is as outlined below.

Methodology

1. Start with low frequency two port model, obtain Av, Ai, G_m at low frequency
2. Identify the nodes (S/D/G/B for MOS; B/E/C for BJT) and add in capacitance in active device
3. Use Miller Approximation in conjunction with OCT to estimate bandwidth ($w_{3\text{dB}}$).

   Advantage: can directly use the "R_in","R_out" from two-port model, only need Av, Ai orG_m much easier.

So taking the C-C Amplifier as an example, the two port model is:

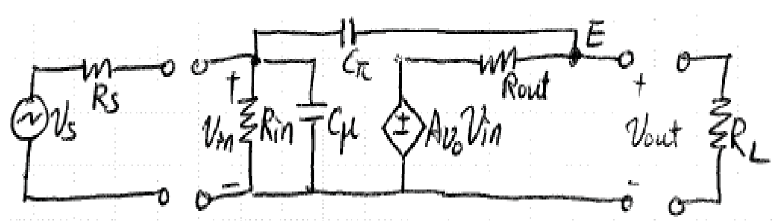


$$
\begin{aligned}
R_{\mathrm{in}} &= \gamma_\pi + \beta_{\mathrm{o}}(\gamma_{\mathrm{o}}||\gamma_{\mathrm{oc}}||R_{\mathrm{L}}) \\
R_{\mathrm{out}} &= \frac{1}{g_{\mathrm{m}}} + \frac{R_{\mathrm{s}}}{\beta_{\mathrm{o}}} \\
A_{\mathrm{vo}} &= 1 \\
A_{\mathrm{v,LF}} &= \frac{V_{\mathrm{out}}}{V_{\mathrm{s}}} = \frac{R_{\mathrm{in}}}{R_{\mathrm{s}} + R_{\mathrm{in}}} \cdot (1) \cdot \frac{R_{\mathrm{L}}}{R_{\mathrm{L}} + R_{\mathrm{out}}}
\end{aligned}
$$

Large $g_{\mathrm{m}}, \beta_{\mathrm{o}}$ will give desired resistances for voltage buffer. High R_{in}, low R_{out}

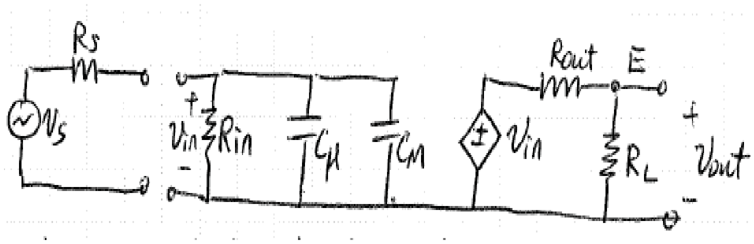
$$
= \frac{\gamma_\pi + \beta_{\mathrm{o}}(\gamma_{\mathrm{oc}}||\gamma_{\mathrm{o}}||R_{\mathrm{L}})}{R_{\mathrm{L}} + \gamma_\pi + \beta_{\mathrm{o}}(\gamma_{\mathrm{oc}}||\gamma_{\mathrm{o}}||R_{\mathrm{L}})}(1)\frac{R_{\mathrm{L}}}{R_{\mathrm{L}} + \frac{1}{g_{\mathrm{m}}} + \frac{R_{\mathrm{s}}}{}}
$$

Identify the B/E/C and add in capacitances



Note: the other end of C_π is to the right of R_{out}! That is where "E" node is! C_π is in the input/output feedback position.

Use Miller Approximation:



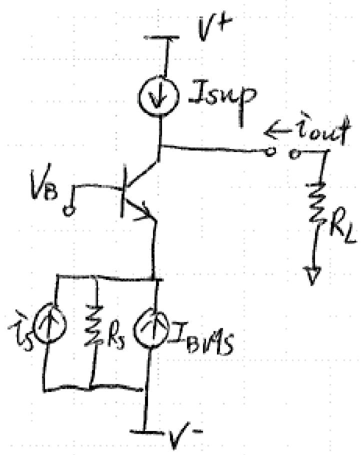
where $C_M = C_\pi(1 - A_{v_{C_\pi}})$. $A_{v_{C_\pi}}$ is the voltage gain across C_π (*not* across overall amplifier).
What is the voltage gain across C_π? $\frac{V_{out}}{V_{in}}$ instead of $\frac{V_{out}}{V_s}$
Or, it is $\frac{V_{out}}{V_s}$ when $R_s = 0$.

$$\begin{aligned}
\frac{V_{out}}{V_{in}} &= \frac{V_{out}}{V_s}\Big|_{R_s=0} = \frac{\gamma_\pi + \beta_o \overbrace{R_L}^{\text{typically } \gamma_o||\gamma_{oc} \gg R_L}}{\cancel{R_s} + \gamma_\pi + \beta_o R_L}(1)\frac{R_L}{R_L + \frac{1}{g_m} + \frac{\cancel{R_s}}{\cancel{\beta_o}}} \\
\therefore A_{v_{C_\pi}} &= \frac{R_L}{R_L + 1/g_m} \\
C_M &= C_\pi(1 - A_{v_{C_\pi}}) = C_\pi\left(\frac{1/g_m}{R_L + 1/g_m}\right) = C_\pi\left(\frac{1}{1 + g_m R_L}\right)
\end{aligned}$$

If $\frac{1}{g_m} \ll R_L$, then $A_{v_{C_\pi}} \longrightarrow 1 \quad \Longrightarrow \quad C_M \longrightarrow 0 \quad w_{3dB} = \frac{1}{(R_s||R_{in}) \cdot (C_\mu + C_M)}$
In contrast to C-S or C-E amplifier, the Miller effect reduces the capacitance in this case, which will give better frequency response: (or another way to look at it, effect of C_π is very small, since voltage gain across C_π is ≈ 1. We do not need a lot of charges to go in/out the capacitor. And typically the movement of charges is the source to slow down the frequency response).

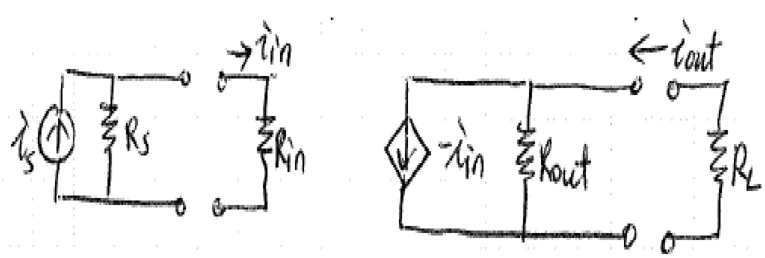
- **Therefore like C-D, Miller effect reduces capacitor value, $\Longrightarrow$ expect good frequency response.**
- **Use of C-C: for multistage amplifiers, can enable high R_{in}, low R_{out}, won't degrade frequency response**

Common-Base Amplifier



Current buffer:

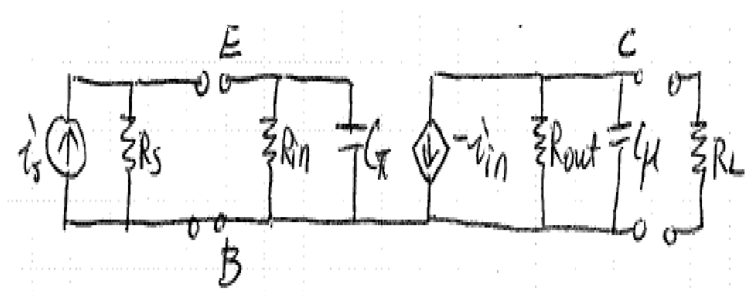
1. Two port model (for current amplifier)



Low frequency current gain

$$
\begin{aligned}
\frac{i_{\text{out}}}{v_{\text{s}}} &= \frac{R_{\text{s}}}{R_{\text{s}} + R_{\text{in}}}(-1)\frac{R_{\text{out}}}{R_{\text{out}} + R_{\text{L}}} \\
R_{\text{in}} &= \frac{1}{g_{\text{m}}} \\
R_{\text{out}} &= \gamma_{\text{oc}}||\left[\gamma_{\text{o}}(1 + g_{\text{m}}(\gamma_{\pi}||R_{\text{s}}))\right]
\end{aligned}
$$

2. Label B/E/C, add in capacitances



No capacitor in the feedback position $\Longrightarrow$ Do not need Miller Approximation. Use OCT

- $C_\pi : R_{\mathrm{TH}_{C_\pi}} = R_s || R_{in} = R_s || \frac{1}{g_m}$
- $C_\mu : R_{\mathrm{TH}_{C_\mu}} = R_{out} || R_L = R_L || (\gamma_{oc} || (\gamma_o + g_m \gamma_o (\gamma_\pi || R_s)))$

Let us try to make some simplifications (if conditions are met) for a on w_{3dB}:

If R_s not so small, since $\frac{1}{g_m}$ is small ($\sim 100\Omega$),

$$
\begin{aligned}
R_s || \frac{1}{g_m} &\simeq \frac{1}{g_m} \Longrightarrow T_{C_\pi} = \frac{C_\pi}{g_m} \\
\text{And if } R_s &\gg \gamma_\pi (\sim 10\,\mathrm{k}\Omega) \\
R_{out} &= \gamma_{oc} || (\gamma_o + g_m \gamma_o (\gamma_\pi || R_s)) \\
g_m \gamma_o (\gamma_\pi || R_s)) &= g_m \gamma_\pi \gamma_o = \beta_o \gamma_o \\
R_{out} &\longrightarrow \gamma_{oc} || \beta_o \gamma_o \text{ can be quite large} \\
\Longrightarrow R_{\mathrm{TH}_{C_\mu}} &\approx R_L || (\gamma_{oc} || \beta_o \gamma_o) \simeq R_L \\
w_{3dB} &\simeq \frac{1}{\underline{C_\pi} + C_\mu R_L} \text{ can be approaching } w_T = \left(\frac{g_m}{C_\pi + C_\mu} \right) \text{ - a good current buffer}
\end{aligned}
$$