Recitation 2: Equilibrium Electron and Hole Concentration from Doping

Here is a list of new things we learned yesterday:

1. Electrons and Holes
2. Generation and Recombination
3. Thermal Equilibrium
4. Law of Mass Action
5. Doping - (donors and acceptors) and charge neutrality
6. Intrinsic Semiconductor vs. Extrinsic Semiconductor
7. Majority and Minority carriers

1 Electrons and Holes

This refers to the "free" electrons and holes. They carry charges (electron -ve and hole +ve), and are responsible for electrical current in the semiconductor. Concentration of electron (= n) and hole (= p) is measured in the unit of /cm^{+3} or cm^{-3} (per cubic centimeter). Remember in Si the atomic density is $5 \times 10^{22}\,\mathrm{cm}^{-3}$, very useful number

2 Generation and Recombination

Generation is one way to obtain "free" e & h in semiconductors. 1 electron-hole pair (1e + 1h) is generated by breaking a bond. Recombination is the reverse process.

3 Thermal Equilibrium

A concept which will be used very often. Thermal equilibrium is defined as steady state + no extra energy source. Note that we have generation or recombination under thermal equilibrium. It is just that the two rates are equal and cancel each other, so the concentrations of e & h do not change. n_o and p_o refer to concentrations in thermal equilibrium.

4 Law of Mass Action

At each temperature T, under thermal equilibrium:

$$\begin{aligned} n_\mathrm{o} \cdot p_\mathrm{o} &= \text{constant} = f(T) \ \text{(only depends on temperature)} \\ n_\mathrm{o} \cdot p_\mathrm{o} &= n_\mathrm{i}^2(T) \qquad (n_\mathrm{i} \equiv \text{intrinsic carrier concentration}) \end{aligned}$$

This is like a chemical reaction:

$$\begin{array}{rcll} \mathrm{H_20} & \leftrightharpoons & \mathrm{H^+ + OH^-} & [H^+][OH^-] = 10^{-14}\,(\mathrm{mol/L})^2 \text{ at Room T} \\ \text{bond} & \leftrightharpoons & \mathrm{e^- + h^+} & n_\mathrm{o} \cdot p_\mathrm{o} = n_\mathrm{i}^2(T) = 10^{20}\,(\mathrm{cm^{-3}})^2 \text{ at Room T} \end{array}$$

Note n_i has a temperature dependence:

$$n_\mathrm{i} = A \cdot (T)^{3/2} \mathrm{e}^{-\frac{E_\mathrm{G}}{2k_\mathrm{B}T}}$$

A is a constant, T is in Kelvin, $T(\mathrm{K}) = 273 + \mathrm{T}(^\circ\mathrm{C})$, and $k_\mathrm{B} = 8.62 \times 10^{-5}\,\mathrm{eV/K}$. E_G is the "Bandgap" energy of the semiconductor - it also corresponds to the ease of bond breakage. For Si, $E_\mathrm{G} = 1.12\,\mathrm{eV}$.

Example 1

At room temperature, $T = 300\,\mathrm{K}, \mathrm{n_i}(300\,\mathrm{K}) = 1 \times 10^{10}\,\mathrm{cm^{-3}}$. What is $n_\mathrm{i}(500\,^\circ\mathrm{C})$?

$$n_\mathrm{i}(500\,^\circ\mathrm{C}) = n_\mathrm{i}(773\,\mathrm{K})$$

$$\frac{n_\mathrm{i}(773\,\mathrm{K})}{n_\mathrm{i}(300\,\mathrm{K})} = \left(\frac{773}{300}\right)^{3/2} \times \frac{\mathrm{e}^{-\frac{E_\mathrm{G}}{2k_\mathrm{B}(773)}}}{\mathrm{e}^{-\frac{E_\mathrm{G}}{2k_\mathrm{B}(300)}}} = 4.14 \times \frac{\mathrm{e}^{-8.4}}{\mathrm{e}^{-21.65}} = 3.5 \times 10^6$$

Therefore, $n_\mathrm{i}(773\,\mathrm{K}) = 3.5 \times 10^6 \times \mathrm{n_i}(300\,\mathrm{K}) = 3.5 \times 10^6 \times 10^{10}\,\mathrm{cm^{-3}} = 3.5 \times 10^{16}\,\mathrm{cm^{-3}}$

Something to observe:
At room temperature, $n_\mathrm{o} = p_\mathrm{o} = 10^{10}\,\mathrm{cm^{-3}}$ for Si. Atomic density is $5 \times 10^{22}\,\mathrm{cm^{-3}}$. Therefore, only a tiny fraction of atoms ($\frac{10^{10}}{5 \times 10^{22}} = \frac{1}{5 \times 10^{12}} = 2 \times 10^{-11}\%$) lose an electron in one of their 4 bonds. By heating up to $500\,^\circ\mathrm{C}$, the concentration of free carriers goes up $\sim 10^6$ (1 million) times, but the percentage is still quite low.

5 Doping and Charge Neutrality

Doping

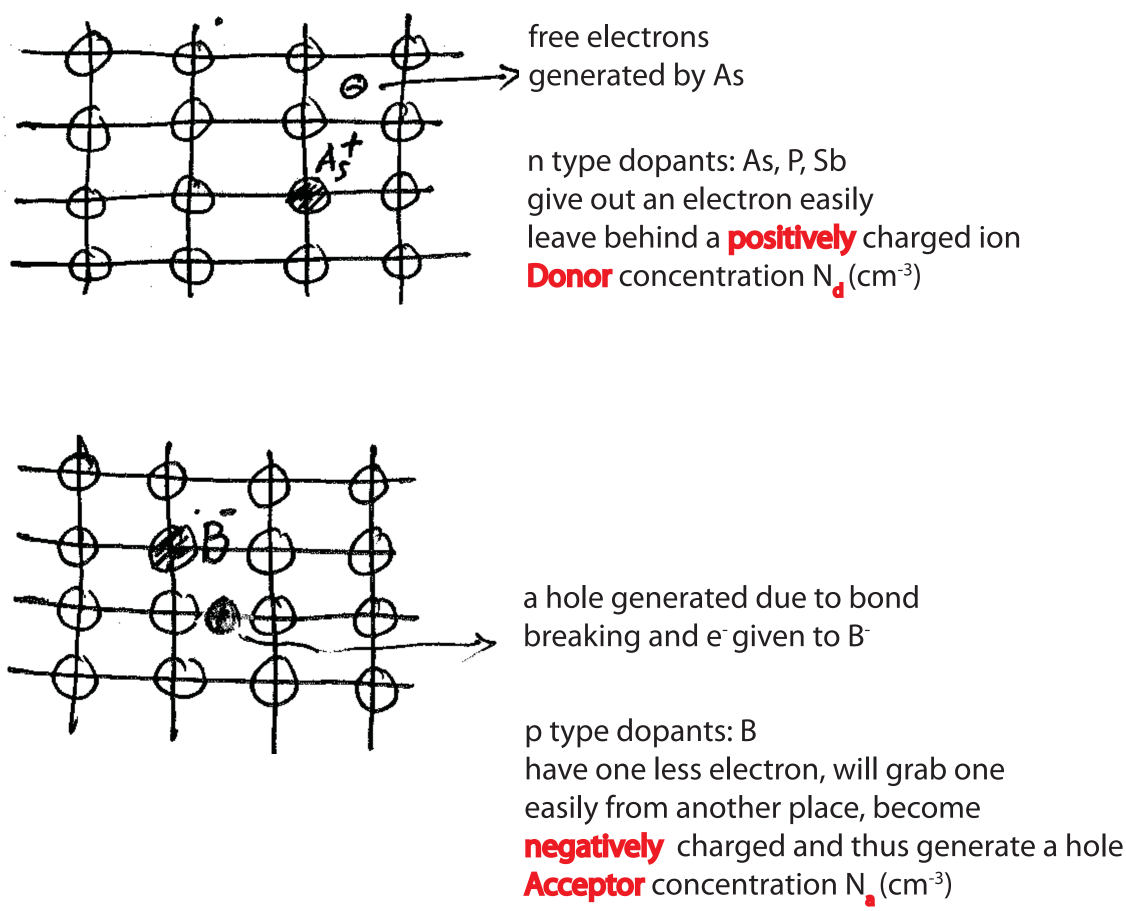

Figure 1: Types of Doping

Charge neutrality

Although foreign atoms are introduced in Si, the overall semiconductor is charge neutral. Therefore, concentration of positive charges = concentration of negative charges. The positive charges include holes (p) and donors (N_{d}). The negative charges include electrons (n) and acceptors (N_{a}).

$$p_{\mathrm{o}} + N_{\mathrm{d}} - n_{\mathrm{o}} - N_{\mathrm{a}} = 0$$

Example 2

Boron doping, dopant concentration $10^{17}\,\mathrm{cm}^{-3}$.
At R.T. under thermal equilibrium N_d =? N_a =? n_o =? p_o =? n_i =? p or n type?

Boron is an acceptor meaning $N_\mathrm{a} = 10^{17}\,\mathrm{cm}^{-3}$, $\mathrm{N_d} = 0$.

$$\begin{aligned}
n_\mathrm{i} &= 10^{10}\,\mathrm{cm}^{-3} \text{ at R.T. under thermal equilibrium (material property, doping does not matter)} \\
p_\mathrm{o} &\simeq N_\mathrm{a} = 10^{17}\,\mathrm{cm}^{-3} \text{ (because } N_\mathrm{a} \gg n_\mathrm{i}\text{)} \\
\because\ n_\mathrm{o} \cdot p_\mathrm{o} &= 10^{20}\,\mathrm{cm}^{-6},\ \mathrm{n_o} = \frac{10^{20}\,\mathrm{cm}^{-6}}{10^{17}\,\mathrm{cm}^{-3}} = 10^{3}\,\mathrm{cm}^{-3}
\end{aligned}$$

6 Intrinsic Semiconductor vs. Extrinsic Semiconductor

In the above example, the semiconductor is **extrinsic** because the carrier concentrations are determined by the dopant concentrations.

Example 3

Si at 500 °C, with As doping $10^{18}\,\mathrm{cm}^{-3}$, extrinsic or intrinsic?

At 500 °C, $\mathrm{n_i}(773\,\mathrm{K}) = 3.5 \times 10^{16}\,\mathrm{cm}^{-3} > \mathrm{N_d}$
It is **intrinsic** semiconductor even though there is doping.

Example 4

A semiconductor can have both dopings. If $N_\mathrm{a} = 10^{15}\,\mathrm{cm}^{-3}$, $\mathrm{N_d} = 10^{19}\,\mathrm{cm}^{-3} \implies$ n-type Si, $n_\mathrm{o} \gg p_\mathrm{o}$ even though we have $N_\mathrm{a} = 10^{15}\,\mathrm{cm}^{-3}$.
When things get complicated, the following relations always work:

$$\begin{aligned}
p_\mathrm{o} + N_\mathrm{d} - n_\mathrm{o} - N_\mathrm{a} &= 0 \\
n_\mathrm{o} \cdot p_\mathrm{o} &= n_\mathrm{i}^2(T)
\end{aligned}$$

Consider example 2, $N_\mathrm{d} = 0\, N_\mathrm{a} = 10^{17}\,\mathrm{cm}^{-3}$. We said $p_\mathrm{o} \simeq N_\mathrm{a} = 10^{17}\,\mathrm{cm}^{-3}$. How accurate is this approximation?

$$\begin{aligned}
n_\mathrm{o} \cdot p_\mathrm{o} &= n_\mathrm{i}^2(T) \\
\Longrightarrow n_\mathrm{o} &= \frac{n_\mathrm{i}^2(T)}{p_\mathrm{o}} \\
\text{plug into} & \quad \text{charge neutrality} \\
p_\mathrm{o} - \frac{n_\mathrm{i}^2(T)}{p_\mathrm{o}} + N_\mathrm{d} - N_\mathrm{a} &= 0 \\
p_\mathrm{o}^2 - N_\mathrm{a} \cdot p_\mathrm{o} - n_\mathrm{i}^2 &= 0 \\
p_\mathrm{o} = \frac{N_\mathrm{a}}{2} \pm \frac{N_\mathrm{a}}{2}\sqrt{1 + \frac{4n_\mathrm{i}^2(T)}{N_\mathrm{a}^2}} & \\
& \quad \text{discard, otherwise } p_\mathrm{o} < 0 \\
\Longrightarrow p_\mathrm{o} &= \frac{N_\mathrm{a}}{2} + \frac{N_\mathrm{a}}{2}\sqrt{1 + \frac{4n_\mathrm{i}^2(T)}{N_\mathrm{a}^2}}
\end{aligned}$$

$\therefore p_\mathrm{o} \simeq N_\mathrm{a}$ is a good approximation since $\frac{4n_\mathrm{i}^2(T)}{N_\mathrm{a}^2} \ll 1$

7 Majority and Minority Carriers

In example 2, $p_\mathrm{o} \simeq N_\mathrm{a} = 10^{17}\,\mathrm{cm}^{-3} \gg \mathrm{n_o} = 10^3\,\mathrm{cm}^{-3}$. Hole is majority carrier and electron is minority carrier.