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Semiconductor Physics

<u>mass-action law</u>* **under T.E. (or quasi-equilibrium)*

$n_o p_o = n_i^2$ where $n_i^2 = 10^{20}\,\text{cm}^{-3}$ at 300K

n-type: $n_o \cong N_d$ & $p_o \cong \frac{n_i^2}{N_d}$

p-type: $p_o \cong N_a$ & $n_o \cong \frac{n_i^2}{N_a}$

<u>drift velocity</u>* **for v < v$_{sat}$ only* ($v_{sat} = 10^7$ cm/s)

$v_{dn} = -\mu_n E$ (electron) $v_{dp} = \mu_p E$ (hole)

<u>current density (drift + diffusion)</u>

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx} \text{ (electron)}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx} \text{ (hole)}$$

<u>Einstein Relation</u>

$\frac{D_p}{\mu_p} = \frac{kT}{q}$ and $\frac{D_n}{\mu_n} = \frac{kT}{q}$

"thermal voltage" : $V_{th} = \frac{kT}{q} = 26mV$

Electrostatics

<u>Gauss' Law / Electrostatic Potential</u>

$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$ $E(x) = -\frac{d\phi}{dx}$

<u>Poisson Equation</u>: $\boxed{\frac{d^2\phi}{dx^2} = -\frac{dE(x)}{dx} = -\frac{\rho(x)}{\varepsilon}}$

<u>Permittivity</u> in vacuum: $\varepsilon_o = 8.85\times10^{-14} F/cm$

in silicon: $\varepsilon_s = 11.7\varepsilon_o$ in SiO$_2$: $\varepsilon_{ox} = 3.9\varepsilon_o$

<u>Boltzmann relations</u>* *(requires $J_{no} = J_{po} = 0$)*

$n_o(x) = n_i e^{q\phi_o(x)/kT}$ $p_o(x) = n_i e^{-q\phi_o(x)/kT}$

reference point: $\phi_o(x) = 0$ when $n_o = n_i$

<u>60mV Rule</u>* *(requires $J_{no} = J_{po} = 0$)*

$$\boxed{\phi_o(x) = 60mV\log\left(\frac{n_o(x)}{10^{10}}\right) = -60mV\log\left(\frac{p_o(x)}{10^{10}}\right)}$$

For electrons: $\phi_n = \frac{kT}{q}\ln\left(\frac{n_o}{n_i}\right)$

For holes: $\phi_p = -\frac{kT}{q}\ln\left(\frac{p_o}{n_i}\right)$

n$^+$ Si: $\phi_{n+} = 550mV$ p$^+$ Si: $\phi_{p+} = -550mV$

pn junction

<u>in thermal equilibrium</u>

charge neutrality → $N_a x_{po} = N_d x_{no}$

built-in potential $\boxed{\phi_B = \phi_n - \phi_p}$

depletion regions

$x_{po} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_a}\right)\left(\frac{N_d}{N_d+N_a}\right)}$ $x_{no} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_d}\right)\left(\frac{N_a}{N_d+N_a}\right)}$

$$X_{do} = x_{po} + x_{no} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{q}\right)\left(\frac{1}{N_a}+\frac{1}{N_d}\right)}$$

n$^+$p junction: $x_{po} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_a}\right)} = X_{do}$

p$^+$n junction: $x_{no} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_d}\right)} = X_{do}$

depletion capacitance (at T.E.)

$$C_{jo} = \frac{\varepsilon_s}{X_{do}} = \frac{\varepsilon_s}{\sqrt{\left(\frac{2\varepsilon_s\phi_B}{q}\right)\left(\frac{1}{N_a}+\frac{1}{N_d}\right)}}$$

<u>under reverse bias</u>

$\phi_j = \phi_B - V_D$ (V$_D$ is negative)

$x_p(V_D) = x_{po}\sqrt{1-\frac{V_D}{\phi_B}}$ $x_n(V_D) = x_{no}\sqrt{1-\frac{V_D}{\phi_B}}$

$X_d(V_D) = X_{do}\sqrt{1-\frac{V_D}{\phi_B}}$

Depletion capacitance (under bias)

$$C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)} = \frac{C_{jo}}{\sqrt{1-\frac{V_D}{\phi_B}}}$$

MOS capacitor (p-substrate)

Built-in voltage

$\phi_B = \phi_g - \phi_p = \phi_{n+} - \phi_p$

Flatband voltage

$V_{FB} = -\phi_B$

Threshold voltage

$$\boxed{V_T = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p}}$$

Body factor [units = V$^{1/2}$]

$\gamma = \frac{1}{C_{ox}}\sqrt{2\varepsilon_s qN_a}$

Small-signal capacitance

$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$ $C_{dep} = \frac{\varepsilon_s}{X_d(V_{GB})}$

In depletion regime:

$$C = \frac{1}{\frac{1}{C_{ox}}+\frac{1}{C_{dep}}}$$

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Summary for MOS Capacitor (n+ gate, p-type Si substrate)

	$V_{GB} \le V_{FB}$	$V_{GB} = V_{FB}$	$V_{FB} \le V_{GB} \le V_T$	$V_{GB} = V_T$	$V_{GB} \ge V_T$
regime	Accumulation	Flatband	Depletion	Threshold	Inversion
Q_G	$C_{ox}(V_{GB} - V_{FB})$	0	$-Q_B = qN_a X_d(V_{GB})$	$-Q_{B\max} = qN_a X_{d\max}$	$C_{ox}(V_{GB} - V_T) - Q_{B\max}$
Q_S	$-C_{ox}(V_{GB} - V_{FB})$	0	$Q_B = -qN_a X_d(V_{GB})$	$Q_{B\max} = -qN_a X_{d\max}$	$-C_{ox}(V_{GB} - V_T) + Q_{B\max}$
X_d	N/A	N/A	$\frac{\varepsilon_s}{C_{ox}}\left(\sqrt{1 + \frac{4(\phi_B + V_{GB})}{\gamma^2}} - 1\right)$	$X_{d\max} = \sqrt{\frac{2\varepsilon_s(-2\phi_p)}{qN_a}}$	$X_{d\max} = \sqrt{\frac{2\varepsilon_s(-2\phi_p)}{qN_a}}$
ϕ_s	$\approx \phi_p$	ϕ_p	$\phi_p + \frac{qN_a X_d^{\,2}}{2\varepsilon_s}$	$-\phi_p$	$\approx -\phi_p$
C	C_{ox}	C_{ox}	$C = \frac{1}{\frac{1}{C_{ox}} + \frac{X_d}{\varepsilon_s}}$	$C = \frac{1}{\frac{1}{C_{ox}} + \frac{X_{d\max}}{\varepsilon_s}}$	C_{ox}

*Note: in accumulation, gate charge is actually negative, charge in semiconductor is positive (because $V_{GB}<V_{FB}$).