MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.011 Introduction to Communication, Control and Signal Processing
Spring 2004

FIRST EVENING EXAM
Wednesday, March 10th, 7:30 PM – 9:30 PM

- This is a closed book exam, but two $8\frac{1}{2}''\times 11''$ sheets of notes (both sides) are allowed. They can be as big as $8\frac{1}{2}''\times 11''$ or as small as you'd like and you can write on one side or two sides of each, but only two sheets are allowed.
- Everything on the notes must be in your original handwriting (i.e., material cannot be xeroxed from solutions, tables, books, etc).
- You have two hours for this exam.
- Calculators are NOT allowed.
- We will NOT provide a table of transforms.
- There are 4 problems on the exam with the percentage for each part and the total percentage for each problem as indicated. Note that the problems do not all have the same total percentage.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.
- Please be neat—we cannot grade what we cannot decipher.
- We tried to provide ample space for you to write in. However, the amount of space provided is not an indication of the length of the explanation required. Short, to the point explanations are preferred to long ones that show little understanding.

All work and answers must be in the space provided on the exam booklet. You are welcome to use scratch pages that we provide but when you hand in the exam we will not accept any pages other than the exam booklet. There will be absolutely no exceptions.

Exam Grading

In grading all of the 6.011 exams we will be focusing on your level of understanding of the material associated with each problem. When we grade each part of a problem we will do our best to assess, from your work, your level of understanding. On each part of an exam question we will also indicate the percentage of the total exam grade represented by that part, and your numerical score on the exam will then be calculated accordingly.

Our assessment of your level of understanding will be based upon what is given in your solution. An correct final answer with no explanation will not receive full credit, and may even receive no credit at all. An incorrect final answer having a solution and explanation that shows excellent understanding quite likely will receive full (or close to full) credit.

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No work on this page will be evaluated.

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FIRST EVENING EXAM

Wednesday, March 10 , 2004

Full Name:____

	Points	Grader
1(a)		
1(b)		
1(c)		
1(d)		
2		
3(a)		
3(b)		
3(c)		
3(d)		
3(e)		
4(a)		
4(b)		
4(c)		
Total		

Problem 1 (29%)

$x(t)$ is a WSS random process with autocorrelation function $R_{xx}(\tau)$ given by:

$$R_{xx}(\tau) = \frac{\sin(\pi\tau)}{\pi\tau}$$

Consider the random processes $y(t)$ defined in terms of $x(t)$ as:

$$y(t) = x(t) \cdot \cos(2\pi \cdot 10^3 t)$$

(a) (5%) Specify whether or not $y(t)$ is WSS. Clearly justify your answer in one sentence.

IT IS WSS **IT IS NOT WSS**

Explanation:

(b) (8%) $y(t)$ is processed with a C/D converter as indicated in Figure 1-1.

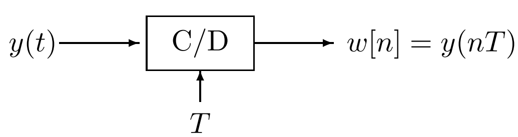

Figure 1-1:

If possible, determine a non-zero value for T so that the discrete-time sequence $w[n]$ in Fig.1-1 is wide-sense stationary. If no such value exists clearly explain why not.

NOT POSSIBLE **POSSIBLE** $T =$

Work to be looked at:

(c) (8%) If possible, determine a value of T so that $w[n]$ is wide-sense stationary and is white, i.e. its PSD $S_{ww}(e^{j\Omega})$ is a non-zero constant for all Ω. If no such value exists clearly explain why not.

NOT POSSIBLE **POSSIBLE** $T =$

Work to be looked at:

(d) (8%) Determine $E\{w^2[n]\}$ in terms of T, i.e. do <u>not</u> assume a specific value of T.

$E\{w^2[n]\} =$

Work to be looked at:

Problem 2 (10%)

$x(t)$ and $y(t)$ are two WSS random processes. The autocorrelation function of $x(t)$ is $R_{xx}(\tau) = e^{-|\tau|}$. State whether or not it is possible to specify a choice for $y(t)$ so that the cross-power-spectral density $S_{xy}(j\omega)$ is as shown in Figure 2-1

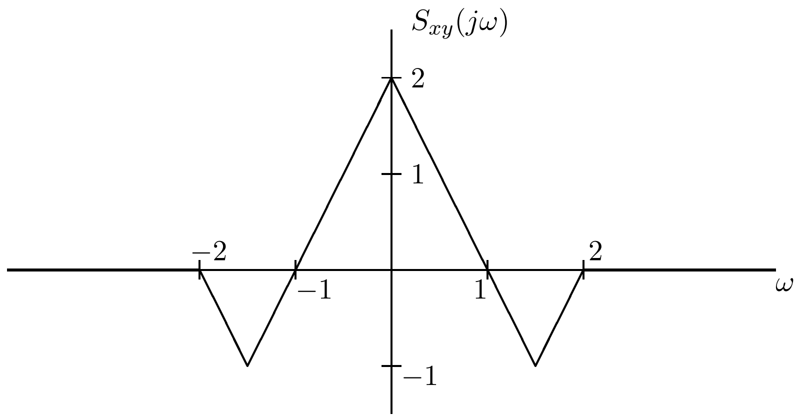

Figure 2-1:

If your answer is no, explain why not and state whether it would be possible with a different choice for $S_{xy}(j\omega)$. If your answer is yes, explain how you might specify or construct $y(t)$.

YES **NO**

Explanation and Reasoning:

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No work on this page will be evaluated.

Problem 3 (31%)

Consider a communication system for which the channel gain $g[n]$ is time-varying, see Figure 3-1:

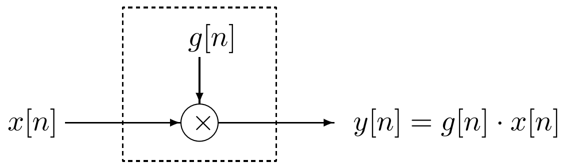


Figure 3-1:

- The transmitted signal $x[n]$ is a wide-sense stationary, zero mean random process.
- The channel gain $g[n]$ is an i.i.d. process with mean μ_g and variance σ_g^2.
- $x[n]$ and $g[n]$ are statistically independent.

(a) (5%) Determine $E\{y[n]\}$

$E\{y[n]\} =$

Work to be looked at:

(b) (10%) Determine $R_{yy}[m]$ and $R_{xy}[m]$ in terms of $R_{xx}[m]$, μ_g, and σ_g^2.

$R_{yy}[m] =$

$R_{xy}[m] =$

Work to be looked at:

For parts (c), (d) and (e) we would like to process $y[n]$ with an LTI-filter to compensate for the random channel gain as indicated in Figure 3-2.

$$y[n] \longrightarrow \boxed{H(z)} \longrightarrow r[n]$$

Figure 3-2:

where $H(z)$ is chosen so that $r[n]$ is a reasonable estimate of $x[n]$.

(c) (4%) If $\mu_g = 0$ can $H(z)$ be designed so that $r[n]$ is a <u>reasonable</u> estimate of $x[n]$? Explain your answer in one or two sentences.

YES **NO**

Reasoning:

(d) (4%) If μ_g is non-zero and $\sigma_g^2 \rightarrow 0$ can $H(z)$ be designed so that $r[n]$ is a <u>reasonable</u> estimate of $x[n]$? Explain your answer in one or two sentences.

YES **NO**

Reasoning:

(e) (8%)As we will see later in the semester an optimal (in some sense) choice for $H(z)$ is:

$$H_{opt}(z) = \frac{S_{xy}(z)}{S_{yy}(z)}$$

(i) Determine $H_{opt}(z)$ in terms of μ_g, σ_g^2 and $S_{xx}(z)$.

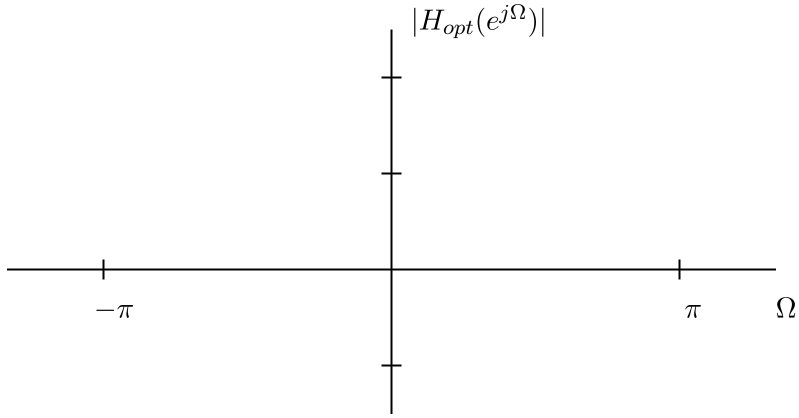
$H_{opt}(z) =$

Work to be looked at:

(ii) Sketch the magnitude of $H_{opt}(e^{j\Omega})$ for the special case $\mu_g = 10$, $\sigma_g^2 = 1$ and

$$S_{xx}(e^{j\Omega}) = \begin{cases} 4, & |\Omega| < \pi/4 \\ 0, & \text{else} \end{cases}$$

as a function of frequency in the graph below. Describe in one or two sentences why this filter is reasonable.



Explanation:

Problem 4 (30%)

Consider a discrete-time system represented by the state-space equations below:

$$\begin{aligned}\mathbf{q}[n+1] &= \mathbf{A}\mathbf{q}[n] + \mathbf{b}x[n] \\ y[n] &= \mathbf{c}^T\mathbf{q}[n]\end{aligned}$$

with $\mathbf{A} = \begin{bmatrix} \frac{3}{2} & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, and $c = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. It is also given that $\mathbf{q}[n] = \mathbf{0}$ for $n \le -1$ and $x[n] = \delta[n+1]$.

(a) (8%) Determine the eigenvalues λ_1, λ_2 and the corresponding eigenvectors $\mathbf{v_1}$, $\mathbf{v_2}$ of the system .

$\lambda_1 =$, $\mathbf{v_1} =$, $\lambda_2 =$, $\mathbf{v_2} =$

Work to be looked at:

(b) (10%) Determine $\mathbf{q}[n]$ for $n \geq 0$

$\mathbf{q}[n] =$

Work to be looked at:

(c) (12%) We want to express the system in terms of the new set of state variables:

$$\mathbf{f}[n] = \begin{bmatrix} f_1[n] \\ f_2[n] \end{bmatrix} = \mathbf{W}\mathbf{q}[\mathbf{n}]$$

In other words, we want to describe the system using the equations:

$$\begin{aligned} \mathbf{f}[n+1] &= \tilde{\mathbf{A}}\mathbf{f}[n] + \tilde{\mathbf{b}}x[n] \\ y[n] &= \tilde{\mathbf{c}}^T\mathbf{f}[n] \end{aligned}$$

State whether or not it is possible to find a transformation matrix $\mathbf{W}$ such that $\tilde{\mathbf{c}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\tilde{\mathbf{b}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. If yes give one possible choice for the transformation matrix $\mathbf{W}$?

NOT POSSIBLE **POSSIBLE**

$\mathbf{W} =$

Work to be looked at: