MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

**6.007 – Applied Electromagnetics**
Spring 2011

**Problem Set 4: Forces and Magnetic Fields**
*Due Friday, March 4, 2011*

## Problem 4.1 – DC Motor

A 'cartoon' view of a DC motor is shown in Figure 4.1.1 [1]. While it is said to be a 'cartoon' view, this is actually not a bad picture of a cross-section of the machine. It consists of a perhaps oddly shaped magnetic circuit on the outside with a 'Yoke' and 'Poles' made of ferromagnetic material with $\mu >> \mu_0$, and a rotor, also made of ferromagnetic material. There are conductors wound in slots in that rotor. In this problem, we are going to do a very quick analysis of this motor.

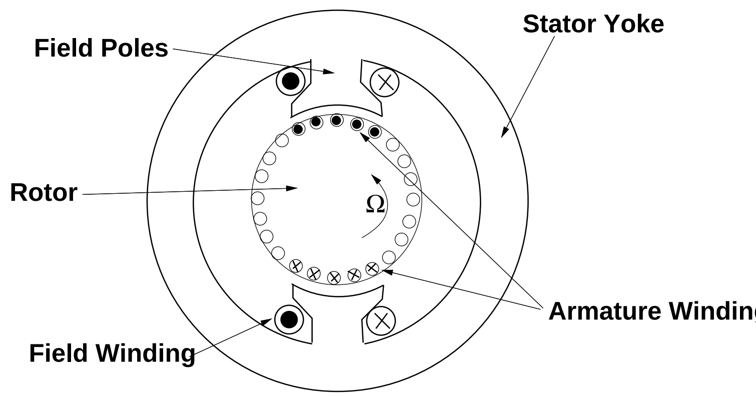



Figure 4.1.1: Cartoon View of a DC motor

**(a)** First, note that because of the ferromagnetic parts of the machine, an Ampere's Law loop that goes vertically through the machine from the bottom pole, the rotor and the top pole and then around through the outer yoke is in ferromagnetic material everywhere but in the two air-gaps, which have radial dimension $g$. Assuming that the field winding wrapped around each pole has $N_f$ turns and that current in that field winding is carrying current $I_f$, find the magnetic flux density $B_r$ crossing the air-gaps. What is the *SIGN* of the radial magnetic flux in the two gaps?

**(b)** Now, assume that there are $N_a$ turns in the armature (this is the rotor in a DC machine). That is, current into the rotor passes through wires in the rotor slots $N_a$ times under each pole. In this view, current is coming out of the paper under the top pole and going into the paper under the bottom pole (well, over, but we use the term 'under the pole' in this context). Using the Lorentz Force Law, calculate the torque produced by current $I_a$ in the armature.

**(c)** Note the magic in this motor is produced by the commutator, which is continually switching the wires so that the armature current is always in the locations shown in Figure 4.1.1. However, the rotor may be turning so the wires are moving with respect to the poles. Now, consider one turn of wire that comes out of the paper on one side of the motor and goes back in exactly 180° away on the other side. As the rotor turns the flux linked by that turn changes, resulting in a voltage. Estimate that voltage for the time when that turn of wire is 'under' the poles, as a function of rotor radius $R$, axial length $L$ (the dimension into the paper), magnetic flux density $B_r$ and rotational speed $\Omega$. Remember that the commutator, as it is re-connecting the wires to make current stay under the poles,

[1] From **Electric Power Principles** by J.L. Kirtley, Wiley, 2010

is also rectifying the voltage produced so it always has the same sign. All of the wires under the pole remain connected in series with each other. You should be able to conclude that Figure 4.1.2 shows an adequate representation for the motor, where the armature resistance $R_a$ is the resistance of the coils of wire in the rotor, in series, plus some additional resistance for the brushes, etc. What is the motor coefficient $G$, in terms of the number of armature terms, the number of field turns, motor radius and length and air-gap?

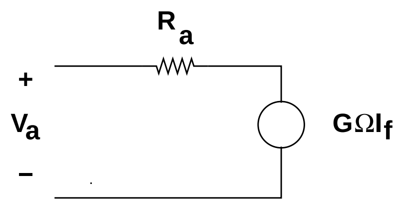



Figure 4.1.2: Equivalent Circuit of a DC motor

**(d)** You should be able to use the first law of thermodynamics to conclude that your expressions for internal voltage $E_a = G\Omega I_f$ and torque produced $T = GI_aI_f$. That is, power into the internal voltage source is the same as mechanical power out.

**(e)** Now, assume the machine is connected to a voltage source with value $V_a$. Calculate, as a function of motor rotational speed $\Omega$, power into the electrical terminals and mechanical power out of the shaft. For high speeds, both of these quantities are negative. What does that mean?

## Problem 4.2 –Energy Method

**(a)** Consider a solenoid with a partially inserted core of magnetic permeability $\mu$, shown in Figure 4.2. The solenoid has a total of $N$ turns, height $h$ and cross sectional area $A$. The solenoid was attached to a power supply sometime in the past, but now the two ends of the coil form a short.

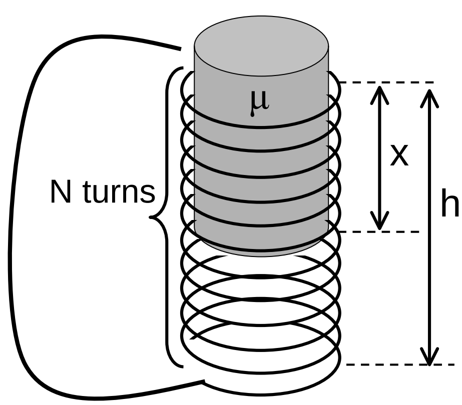



Figure 4.2 Solenoid with partially inserted core

Find an expression for the force on the magnetic core $f_c$ by taking a derivative $\frac{\partial}{\partial x}$ of the stored magnetic energy, and indicate its direction. (Which expression should you use for the stored magnetic energy?) Assume that $\mu \gg \mu_o$ such that only the part containing the magnetic core has significant inductance. Ignore gravity for this problem.

**(b)** Can you solve this problem by using the other expression for stored magnetic energy? Why or why not?

## Problem 4.3 – Junkyard Magnet

Shown in Figure 4.3 a) is a lifting electromagnet similar to what is seen in junkyards. It uses magnetic fields to pick up large pieces of steel, including automobiles. As described in Figure 4.1 b), the magnetic is cylindrical, with a coil of N=1,000 turns contained inside a ferromagnetic material. The radius of the coils is $r_1 = 0.5$ m. The other radius of the ferromagnetic material is $r_2 = 0.8$ m, assuming the coils themselves have negligible thickness. The car (modeled as a slab) and the junkyard magnet both are made with materials with very high permeabilities. The gap between the magnet and the car is assumed to be uniform with value $g$.

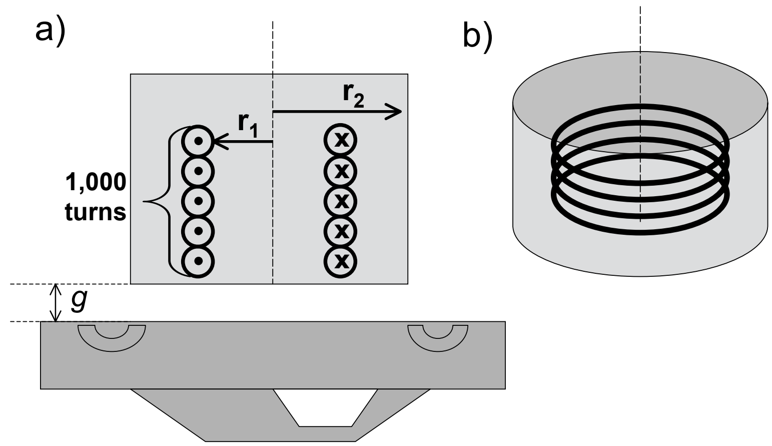



Figure 4.3 Junkyard magnet a) Cross section. b) 3D view

**(a)** Sketch what the magnetic field lines should look like. Draw a magnetic circuit for this situation, indicating which elements of the magnetic circuit correspond with which elements or features of the physical device.

**(b)** Find the reluctance of each element in your magnetic circuit, assuming magnetic permeability is very large for the magnetic material and the car. Compute the inductance of the magnetic as a function of the gap dimension $g$.

**(c)** Compute the force exerted on the car if the current in the coil is 25 Amperes, as a function of the gap dimension $g$.

**(d)** Your answer in part (c) should yield an unreasonable answer for small values of the gap. Why?

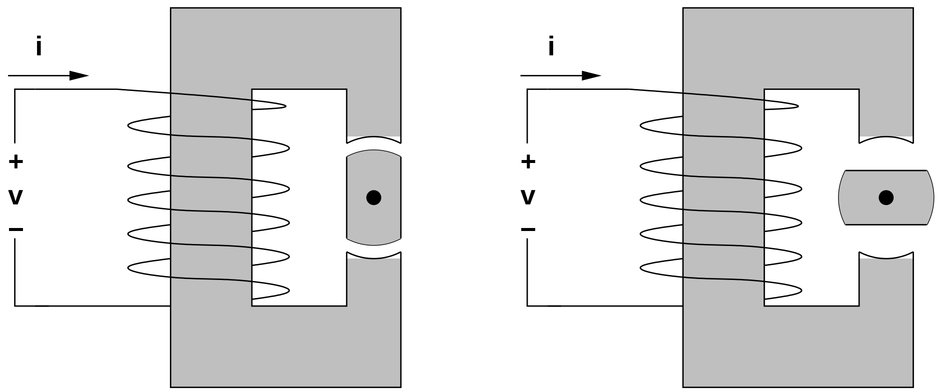
**(e)** To get a more reasonable answer, find the value of magnetic flux density in the gap as a function of current in the coil and the gap $g$. Suppose the system 'saturates' when the magnetic flux density reaches $B = 1.8$ T.

**(f)** Using this result, sketch your best estimate of the actual force as a function of gap for $0 < g < 10$ cm. In doing this, you should assume that magnetic flux goes 'straight across the gap' and that there is no meaningful fringing.

## Problem 4.4 — Variable Reluctance Machine

*This problem tests knowledge of Magnetic Circuits and Motors.*

The variable reluctance machine(VRM) is the simplest AC motor. This VRM consists of a wire with N turns wrapped around a magnet core(with magnetic permeability $\mu >> \mu_0$) that delivers magnetic flux to a gap. Within the gap is a rotating bar or rotor.




(a) $t = t_1$ (b) $t = t_2$

The spinning rotor has length $l$, width $a$ and a cross sectional area of A. At closest approach, the gap $g(t)$ between the rotor and stator is $g_{min}$ when the rotor is aligned and as large as $g_{max}$ when the rotor is out of alignment.

(a) Draw a magnetic circuit for this system. Label which physical portion corresponds to the various reluctances. Determine the reluctance for the gap ($R_{gap}$) and the rotor ($R_{rotor}$) in terms of the geometric and material parameters discussed above.

(b) Determine the maximum and minimum magnetic flux $\phi$ within the gap between rotor and stator. Write your answer in terms of reluctances.

(c) Determine the total inductance $L(\Theta)$ of the motor for the two cases (aligned and unaligned) assuming that $g_{min}$ and $g_{max}$ are known.

(d) Now, assume you can approximate the inductance as a function of angle $\Theta$ to have a sinusoidal variation between maximum and minimum inductance. Sketch that inductance and write an expression for it.

(e) When a constant current is applied, what is the expression for torque as a function of $\Theta$?. You should see that the torque switches sign for half the period, creating a restoring force that causes the motor to oscillate. To keep the motor spinning in one direction., you can turn this into a 'synchronous' motor by driving it with a current that depends on rotor position What form of current would you use? Find the time average torque produced by this machine when driven by your current waveform.