Lecture 1: Introduction and Peak Finding

Lecture Overview

- Administrivia
- Course Overview
- "Peak finding" problem — 1D and 2D versions

Course Overview

This course covers:

- Efficient procedures for solving problems on large inputs (Ex: U.S. Highway Map, Human Genome)
- Scalability
- Classic data structures and elementary algorithms (CLRS text)
- Real implementations in Python
- Fun problem sets!

The course is divided into 8 modules — each of which has a motivating problem and problem set(s) (except for the last module). Tentative module topics and motivating problems are as described below:

1. Algorithmic Thinking: Peak Finding
2. Sorting & Trees: Event Simulation
3. Hashing: Genome Comparison
4. Numerics: RSA Encryption
5. Graphs: Rubik's Cube
6. Shortest Paths: Caltech $\rightarrow$ MIT
7. Dynamic Programming: Image Compression
8. Advanced Topics

Peak Finder

One-dimensional Version

Position 2 is a peak if and only if $b \geq a$ and $b \geq c$. Position 9 is a peak if $i \geq h$.

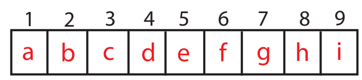
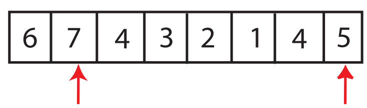


Figure 1: a-i are numbers

<u>Problem</u>: Find a peak if it exists (Does it always exist?)

Straightforward Algorithm



Start from left

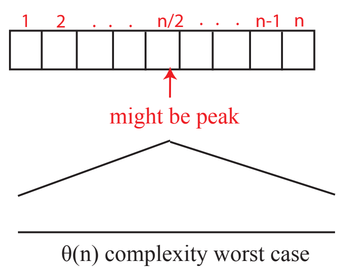
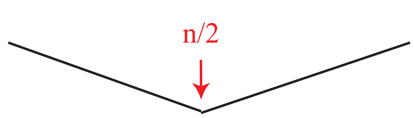


Figure 2: Look at $n/2$ elements on average, could look at n elements in the worst case

What if we start in the middle? For the configuration below, we would look at $n/2$ elements. Would we have to ever look at more than $n/2$ elements if we start in the middle, and choose a direction based on which neighboring element is larger that the middle element?



Can we do better?

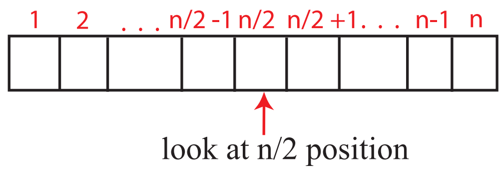


Figure 3: Divide & Conquer

- If $a[n/2] < a[n/2-1]$ then only look at left half $1 \dots n/2 - - - 1$ to look for peak
- Else if $a[n/2] < a[n/2+1]$ then only look at right half $n/2+1 \dots n$ to look for peak
- Else $n/2$ position is a peak: WHY?

$$\begin{aligned} a[n/2] &\geq a[n/2-1] \\ a[n/2] &\geq a[n/2+1] \end{aligned}$$

What is the complexity?

$$T(n) = T(n/2) + \underbrace{\Theta(1)}_{\text{to compare a[n/2] to neighbors}} = \Theta(1) + \dots + \Theta(1) \; (\log_2(n) \; times) = \Theta(\log_2(n))$$

In order to sum up the $\Theta(i)$'s as we do here, we need to find a constant that works for all.
If $n = 1000000$, $\Theta(n)$ algo needs 13 sec in python. If algo is $\Theta(\log n)$ we only need 0.001 sec.
Argue that the algorithm is correct.

Two-dimensional Version

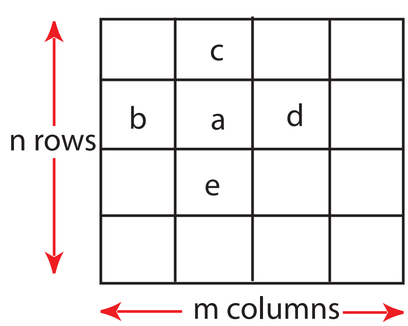


Figure 4: Greedy Ascent Algorithm: $\Theta(nm)$ complexity, $\Theta(n^2)$ algorithm if $m = n$

a is a 2D-peak iff $a \geq b, a \geq d, a \geq c, a \geq e$

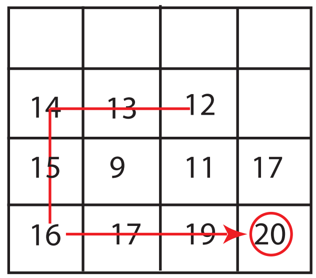
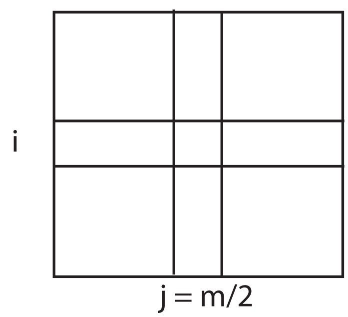


Figure 5: Circled value is peak.

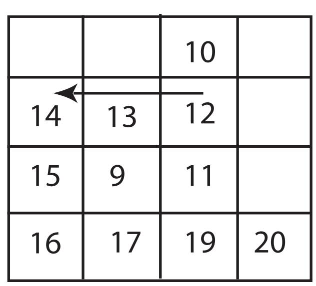
Attempt # 1: Extend 1D Divide and Conquer to 2D



- Pick middle column $j = m/2$.
- Find a 1D-peak at i, j.
- Use (i, j) as a start point on row i to find 1D-peak on row i.

Attempt #1 fails

Problem: 2D-peak may not exist on row i

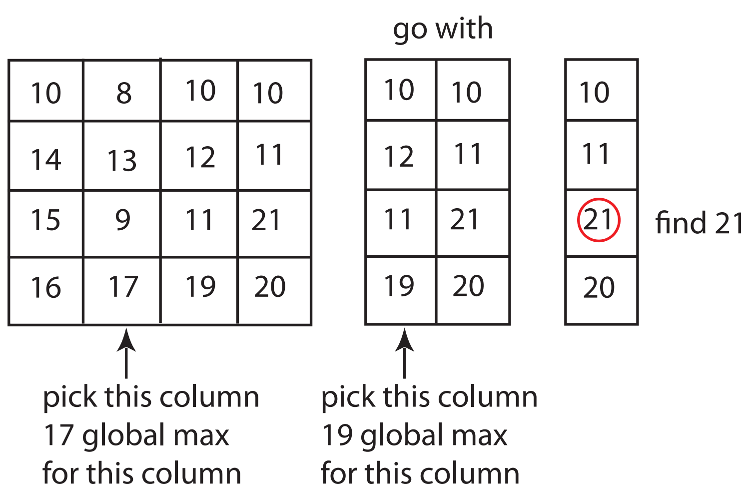


End up with 14 which is not a 2D-peak.

Attempt # 2

- Pick middle column $j = m/2$
- Find global maximum on column j at (i, j)
- Compare $(i, j-1), (i, j), (i, j+1)$
- Pick left columns of $(i, j-1) > (i, j)$
- Similarly for right
- (i, j) is a 2D-peak if neither condition holds $\leftarrow$ WHY?
- Solve the new problem with half the number of columns.
- When you have a single column, find global maximum and you‘re done.

Example of Attempt #2



Complexity of Attempt #2

If $T(n, m)$ denotes work required to solve problem with n rows and m columns

$$
\begin{aligned}
T(n,m) &= T(n, m/2) + \Theta(n) \text{ (to find global maximum on a column — (n rows))} \\
T(n,m) &= \underbrace{\Theta(n) + \ldots + \Theta(n)}_{\log m} \\
&= \Theta(n \log m) = \Theta(n \log n) \text{ if m = n}
\end{aligned}
$$

Question: What if we replaced global maximum with 1D-peak in Attempt #2? Would that work?