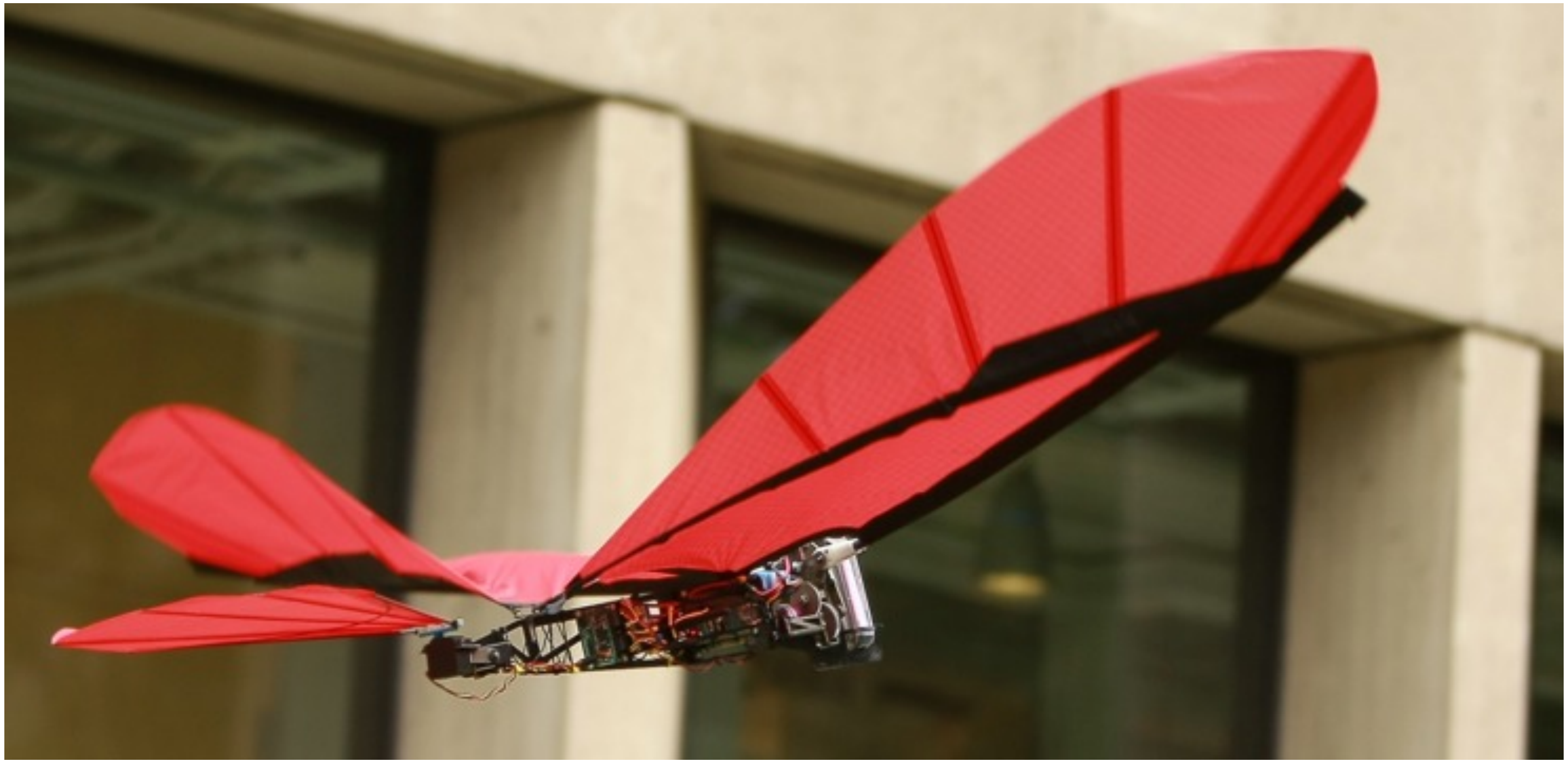
6.003: Signals and Systems

Feedback and Control

October 13, 2011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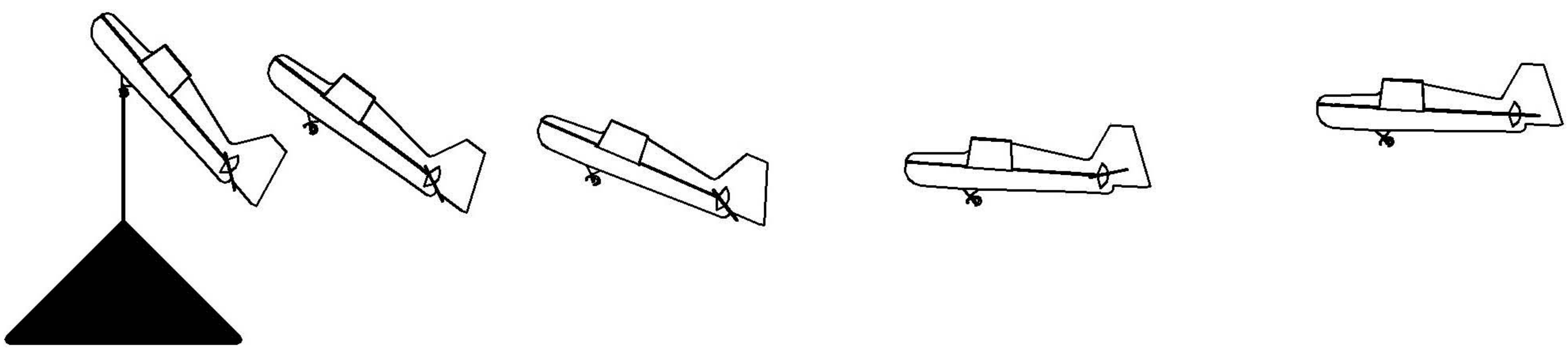
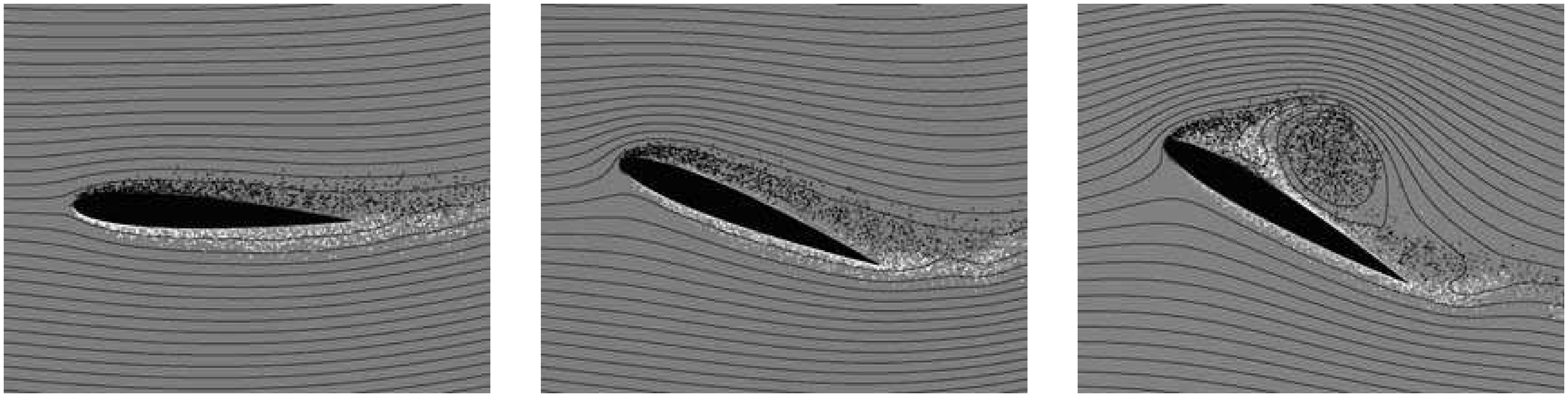
Example: Perching

Can we make a fixed-wing UAV land on a perch like a bird?



The “Perching” Problem



Courtesy of Leon van Dommelen and Szu-Chuan Wang. Used with permission.

Photo from Naval Historical Center Aircraft Data Series.

Photo of a cardinal landing on a branch removed due to copyright restrictions.

Dimensionless Analysis

- Bird or plane...
  - with mass *m*, wing area *S*, operating in a fluid with density ρ
  - which requires a distance *x* to slow from V_0 to V_f
- Distance-averaged drag coefficient, C_D:

$$\langle C_D \rangle = \frac{2m}{\rho S x} \ln\left(\frac{V_0}{V_f}\right)$$

- A few (very preliminary) reference points:

Photo of the Boeing 747-400ER removed due to copyright restrictions.

Vehicle	Average C_D
Boeing 747	0.16
X-31	0.3
Cornell Perching Plane	0.25
Common Pigeon	10

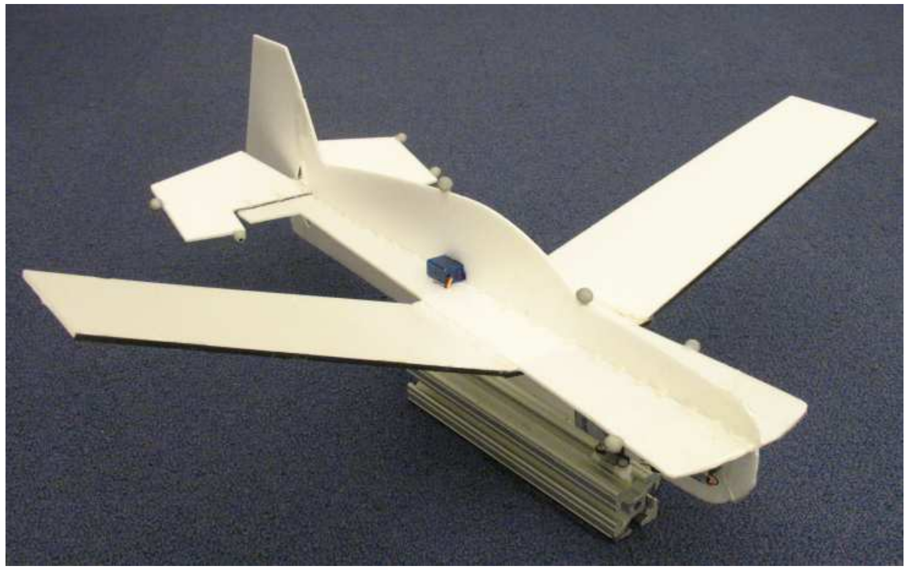
Photos of Cornell perching plane and landing pigeon removed due to copyright restrictions.

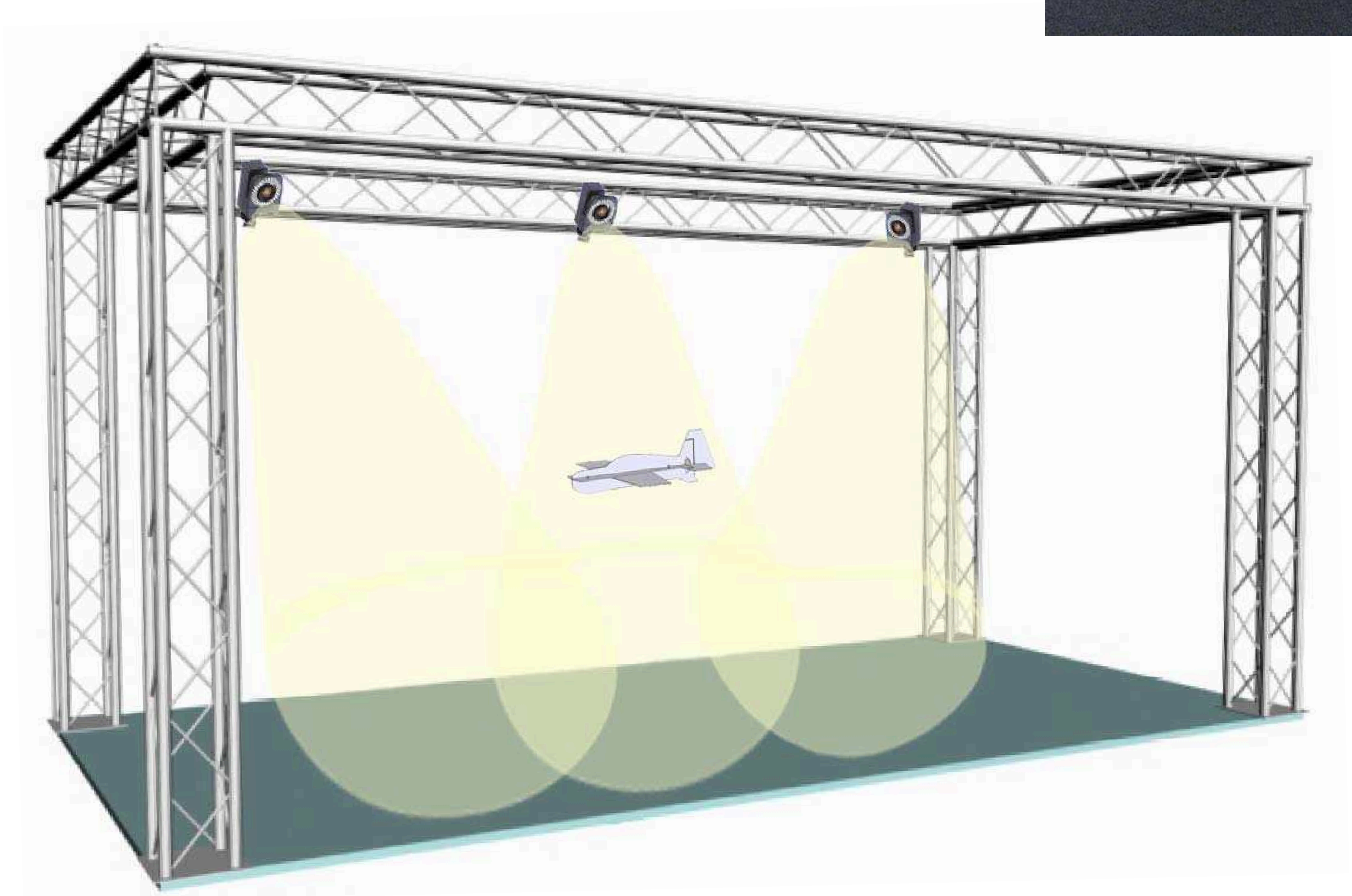
U.S. Navy photo by James Darcy.

Image removed due to copyright restrictions. Please see SlowMoHighSpeed. "Photron SA2 Camera - Eagle Owl in Flight." October 27, 2008. YouTube. Accessed September 25, 2012. http://www.youtube.com/watch?v=LA6XSrM0V_0

Experiment Design

- Glider (no propellor)
- Flat plate wings
- Dihedral (passive roll stability)
- Offboard sensing and control



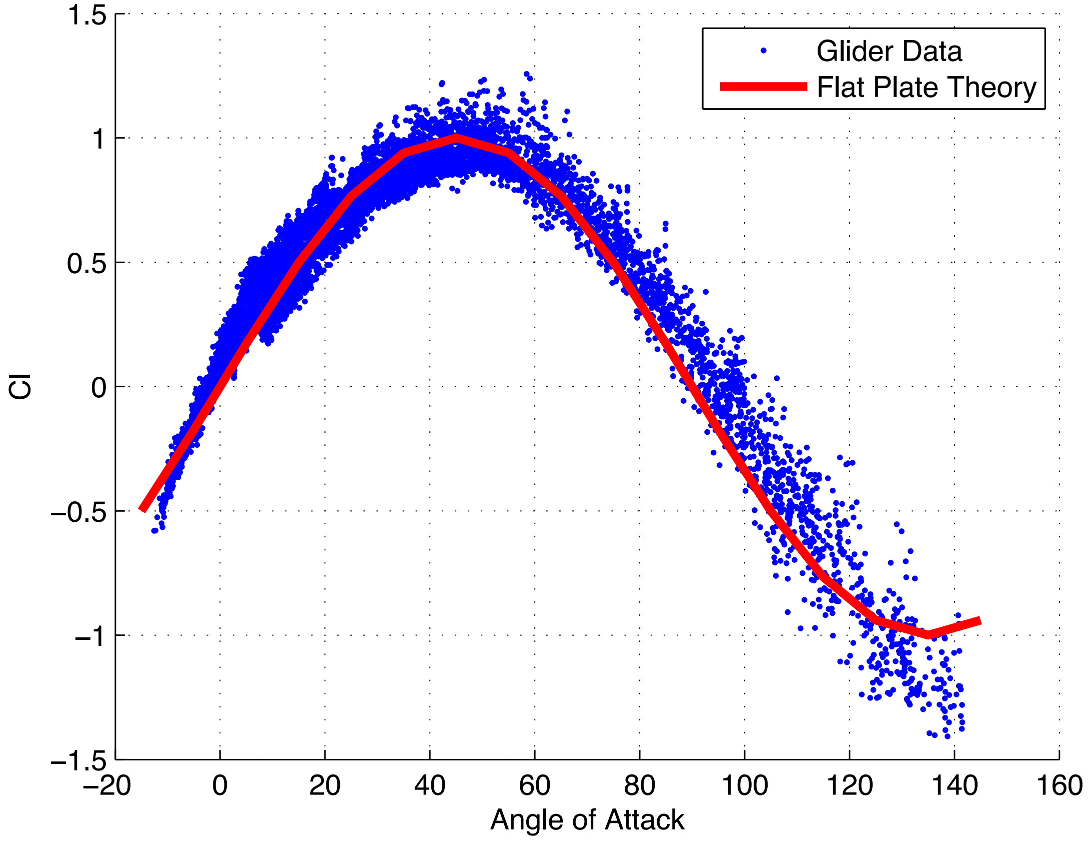


System Identification

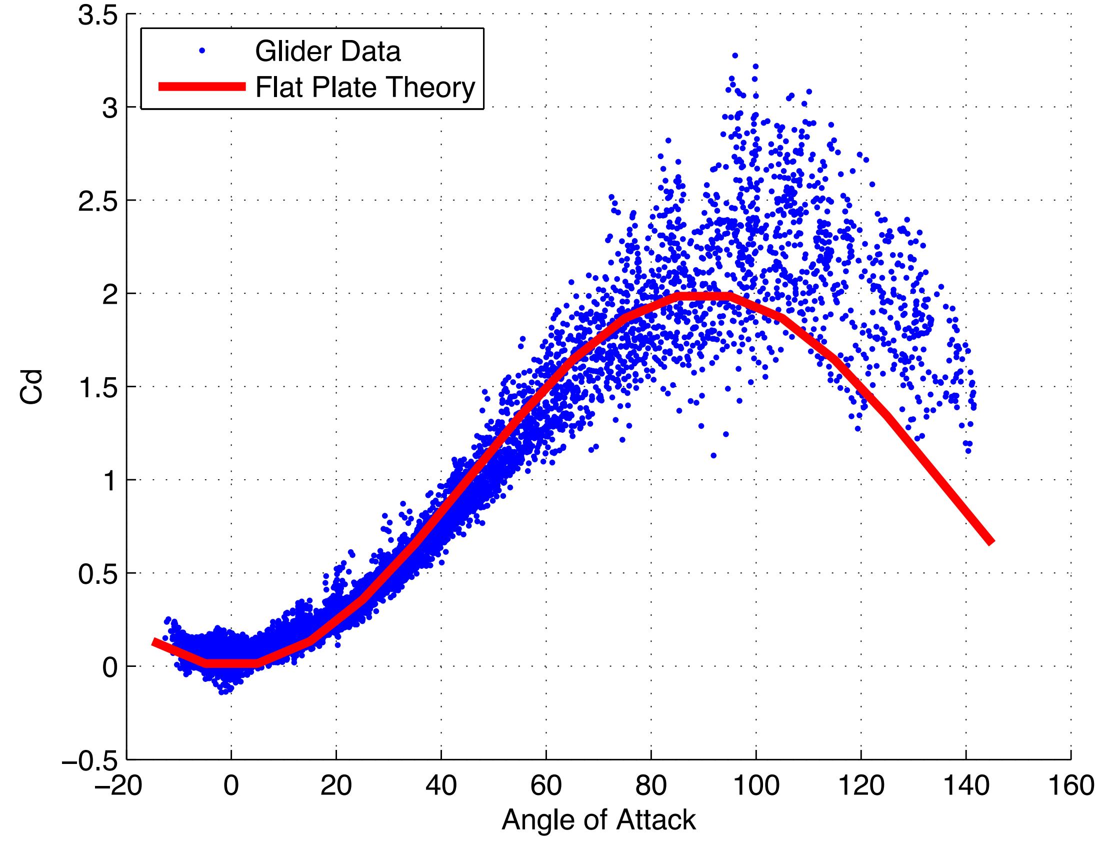
- Nonlinear rigid-body vehicle model
- Linear actuator model (+ saturations, delay)
- Real flight data (no wind tunnel)

System Identification

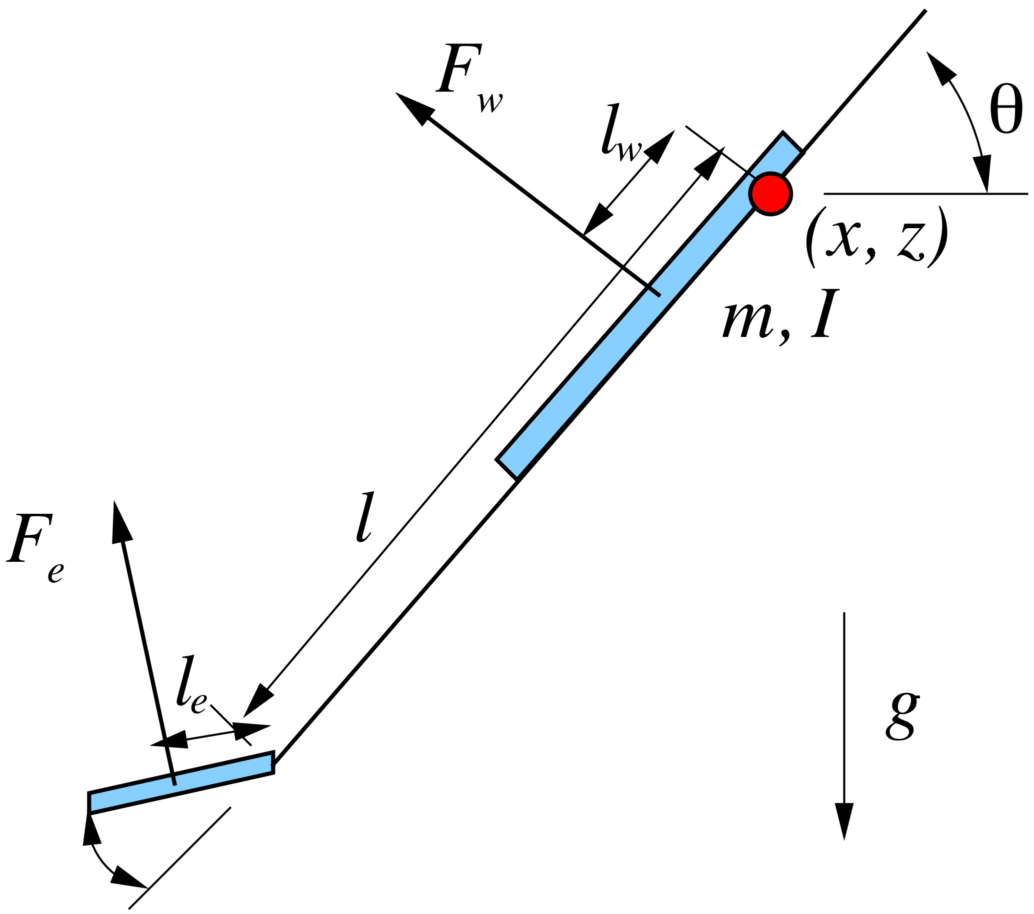
Lift Coefficient



Drag Coefficient



A Dynamic Model



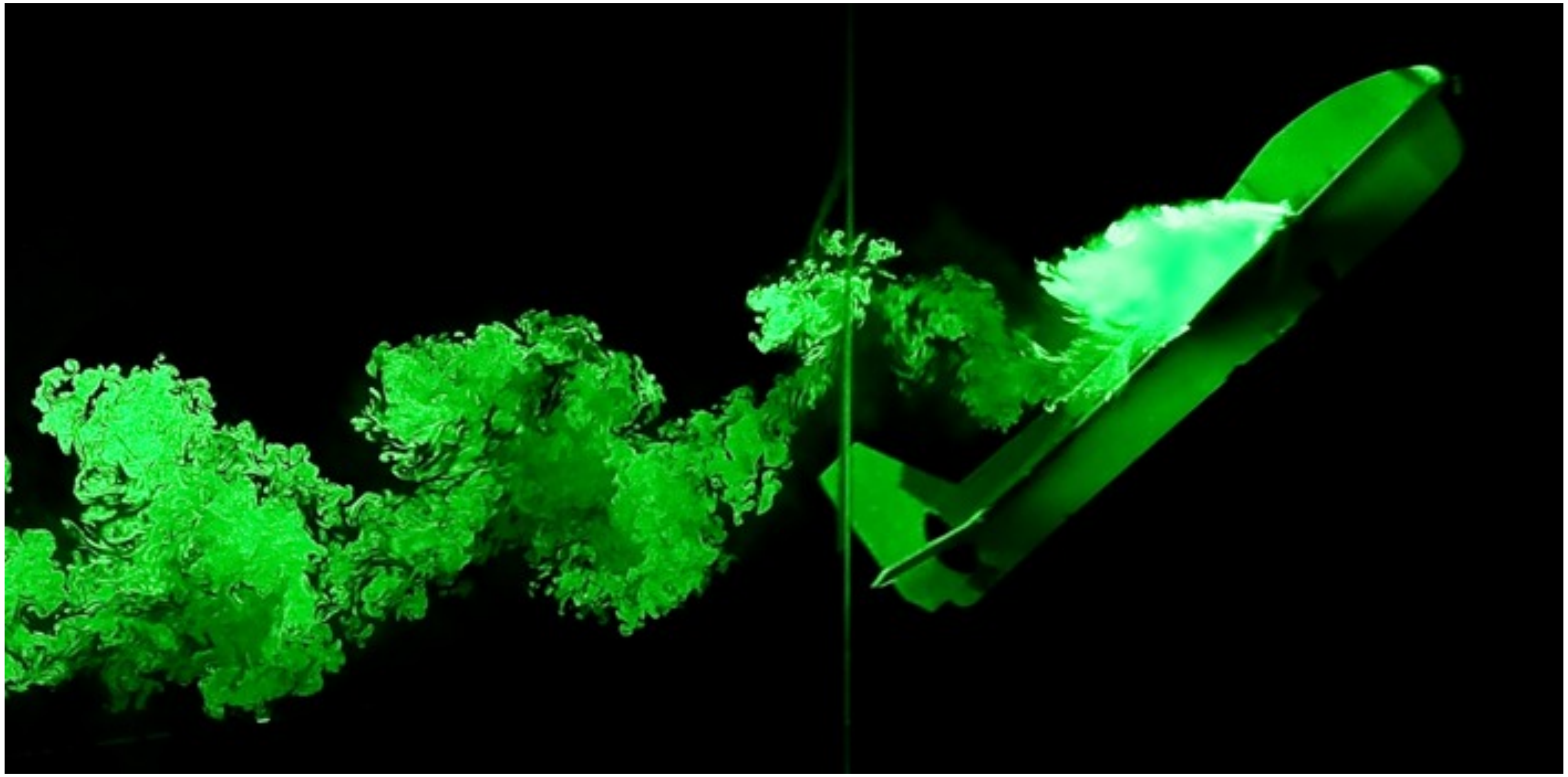
- Planar dynamics
- Aerodynamics fit from data
- State: $\mathbf{x} = [x, y, \theta, \phi, \dot{x}, \dot{y}, \dot{\theta}]$
- Actuator: $\mathbf{u} = \dot{\phi}$

Perching Results

- Enters motion capture @ 6*m/s*
- Perch in < 3.5*m* away
- Entire trajectory < 1*s*

Requires separation!

Flow visualization



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Dimensionless Analysis

Vehicle	Average C_D
Boeing 747	0.16
X-31	0.3
Cornell Perching Plane	0.25
Common Pigeon	10
Our glider	1.1
Cobra maneuver (Mig)	0.9

Feedback is essential...

- to compensate for initial condition errors, disturbances, and imperfect model
- agile airplanes are open loop unstable

open loop

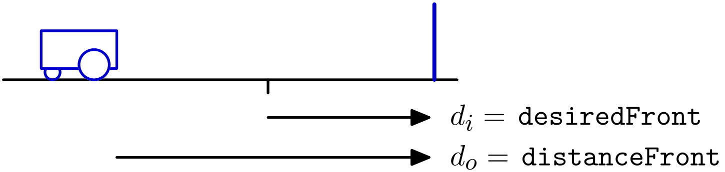
feedback

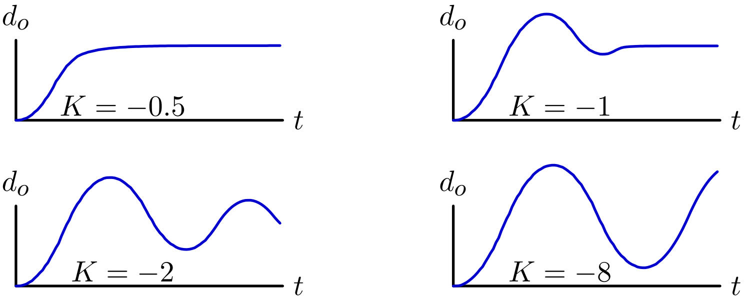
Today's goal

Use systems theory to gain insight into how to control a system.

Example: wallFinder System

Approach a wall, stopping a desired distance d_i in front of it.

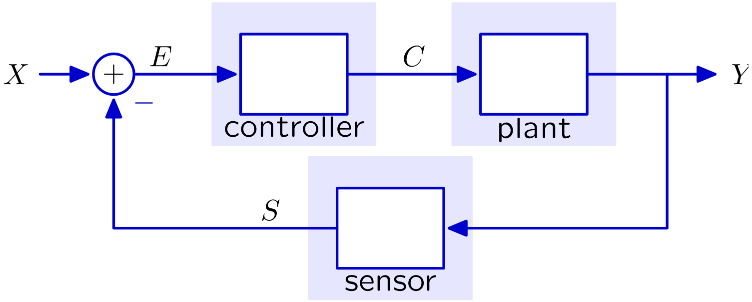




What causes these different types of responses?

Structure of a Control Problem

(Simple) Control systems have three parts.



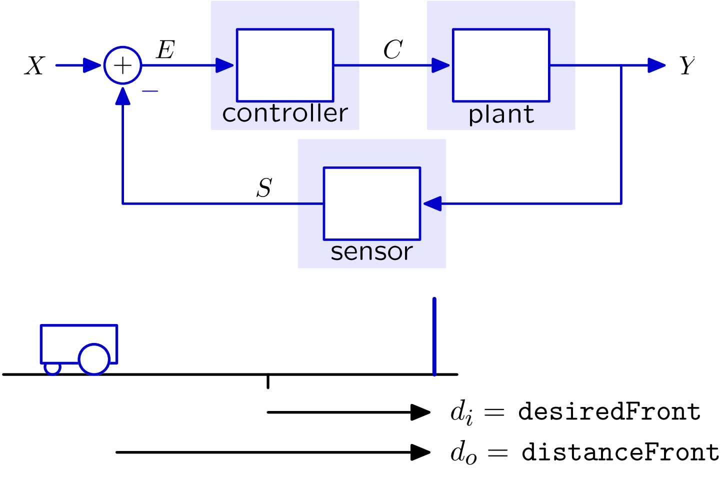
The **plant** is the system to be controlled.

The **sensor** measures the output of the plant.

The **controller** specifies a command C to the plant based on the *difference* between the input X and sensor output S.

Analysis of wallFinder System

Cast wallFinder problem into control structure.



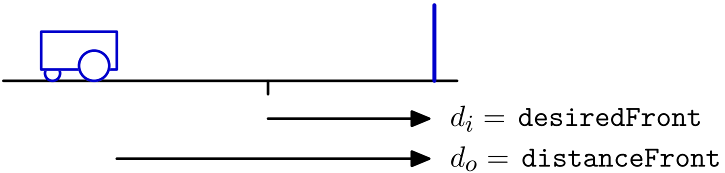
proportional controller: $v[n] = Ke[n] = K\big(d_i[n] - d_s[n]\big)$

locomotion: $d_o[n] = d_o[n-1] - Tv[n-1]$

sensor with no delay: $d_s[n] = d_o[n]$

Analysis of wallFinder System: Block Diagram

Visualize as block diagram.



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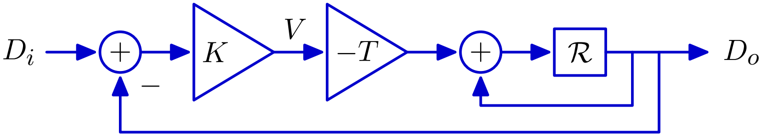
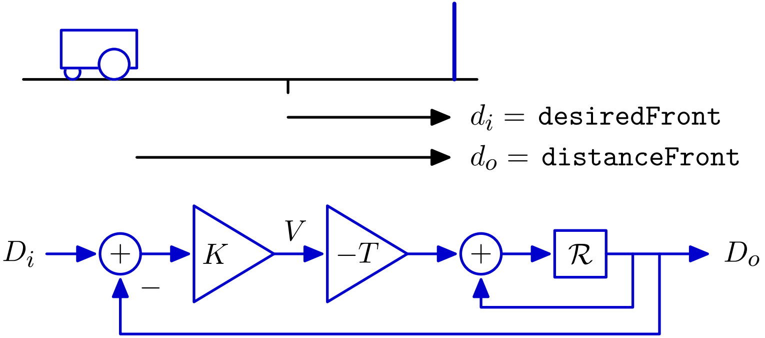


Analysis of wallFinder System: System Function

Solve.



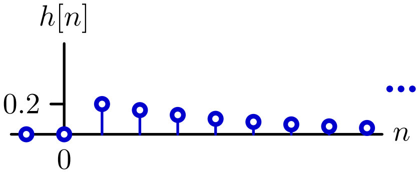
$$\frac{D_o}{D_i} = \frac{\frac{-KT\mathcal{R}}{1-\mathcal{R}}}{1+\frac{-KT\mathcal{R}}{1-\mathcal{R}}} = \frac{-KT\mathcal{R}}{1-\mathcal{R}-KT\mathcal{R}} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}}$$

Analysis of wallFinder System: Poles

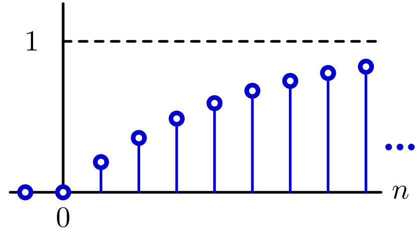
The system function contains a single **pole** at $z = 1 + KT$.

$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}}$$

Unit-sample response for $KT = -0.2$:



Unit-step response $s[n]$ for $KT = -0.2$:



What determines the speed of the response? Could it be faster?

Check Yourself

Find KT for fastest convergence of unit-sample response.

$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}}$$

1. $KT = -2$
2. $KT = -1$
3. $KT = 0$
4. $KT = 1$
5. $KT = 2$

0. none of the above

Check Yourself

Find KT for fastest convergence of unit-sample response.

$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}}$$

If $KT = -1$ then the pole is at $z = 0$.

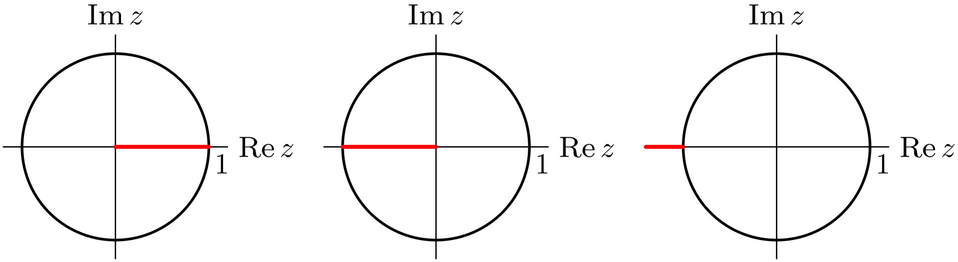
$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}} = \mathcal{R}$$

Unit-sample response has a single non-zero output sample, at $n = 1$.

Analysis of wallFinder System: Poles

The poles of the system function provide insight for choosing K.

$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}} = \frac{(1-p_o)\mathcal{R}}{1-p_o\mathcal{R}} \quad ; \qquad p_0 = 1 + KT$$



$0 < p_0 < 1$	$-1 < p_0 < 0$	$p_0 < -1$
$-1 < KT < 0$	$-2 < KT < -1$	$KT < -2$
monotonic	alternating	alternating
converging	converging	diverging

Check Yourself

Find KT for fastest convergence of unit-sample response.

$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1-(1+KT)\mathcal{R}}$$

1. $KT = -2$
2. $KT = -1$
3. $KT = 0$
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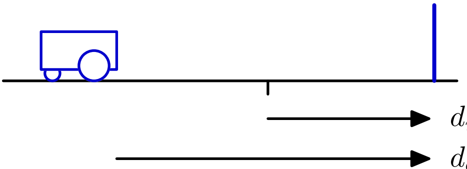
5. $KT = 2$
0. none of the above

Analysis of wallFinder System

The optimum gain K moves robot to desired position in **one** step.



d_i = desiredFront=1 m

d_o = distanceFront=2 m

$$KT = -1$$

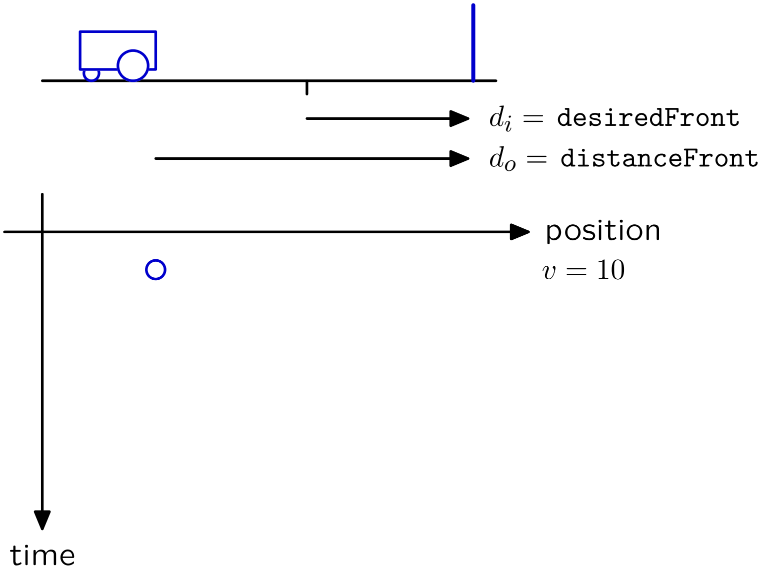
$$K = -\frac{1}{T} = -\frac{1}{1/10} = -10$$

$$v[n] = K\big(d_i[n] - d_o[n]\big) = -10\big(1 - 2\big) = 10 \text{ m/s}$$

exactly the right speed to get there in one step!

Analyzing wallFinder: Space-Time Diagram

The optimum gain K moves robot to desired position in **one** step.



Analyzing wallFinder: Space-Time Diagram

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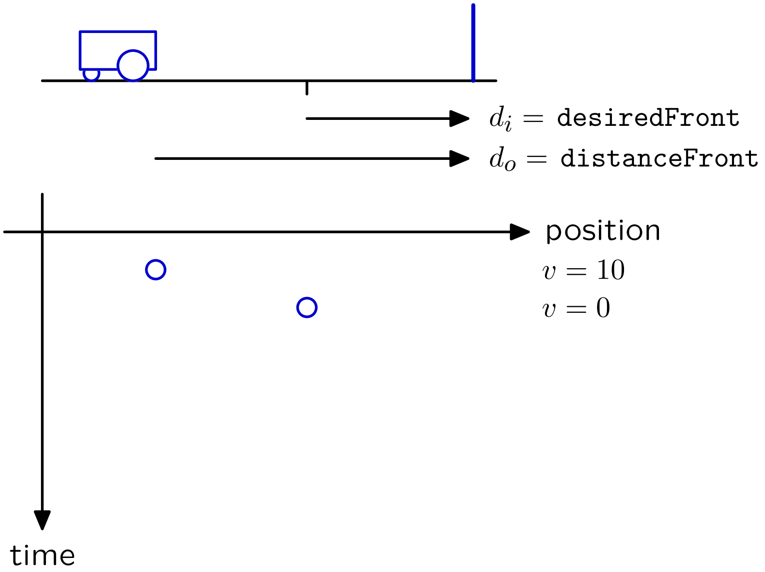

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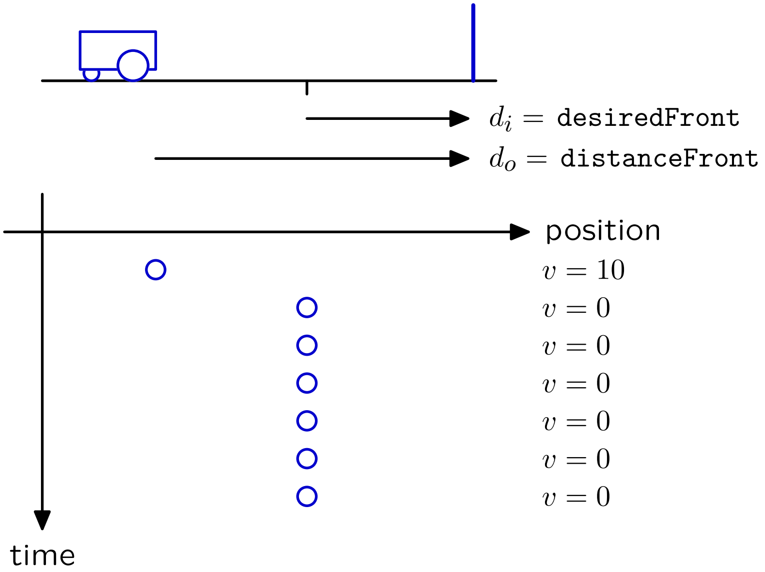
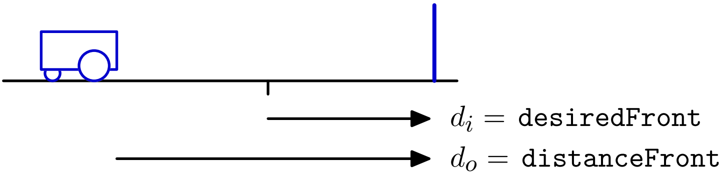


Analysis of wallFinder System: Adding Sensor Delay

Adding delay tends to destabilize control systems.



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locomotion: $d_o[n] = d_o[n-1] - Tv[n-1]$

sensor **with delay**: $d_s[n] = d_o[\mathbf{n-1}]$

Analysis of wallFinder System: Adding Sensor Delay

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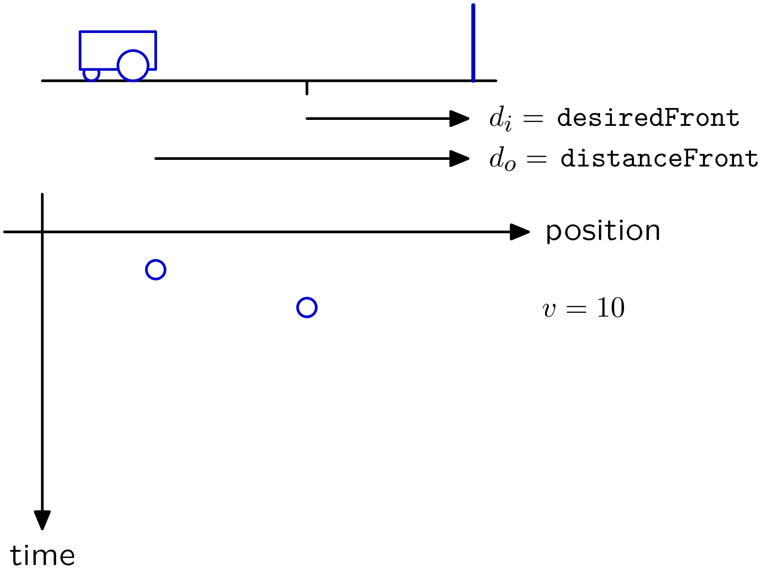

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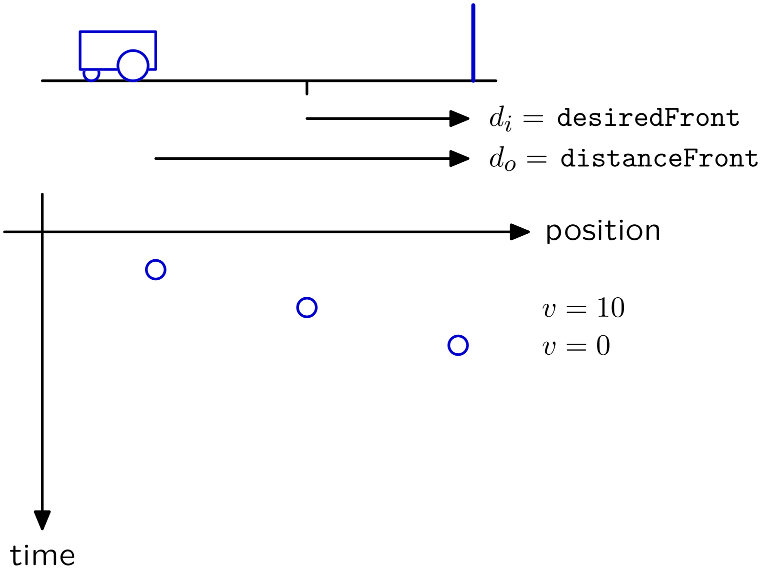

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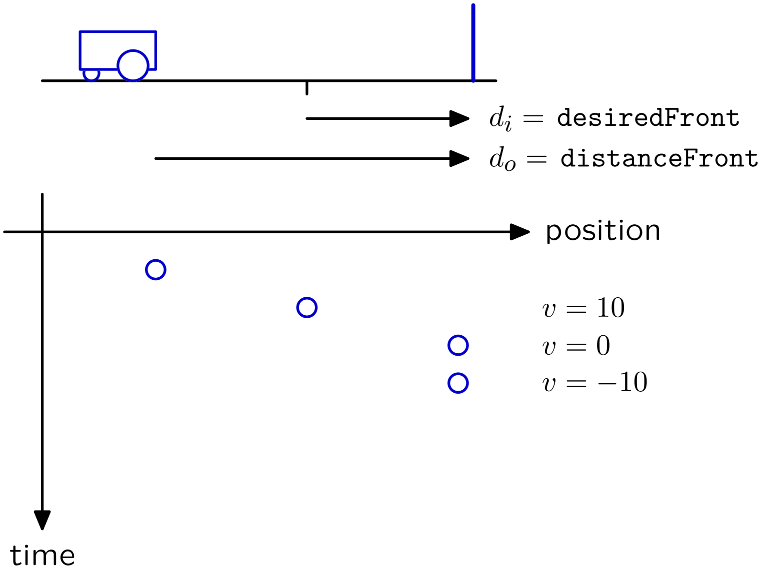

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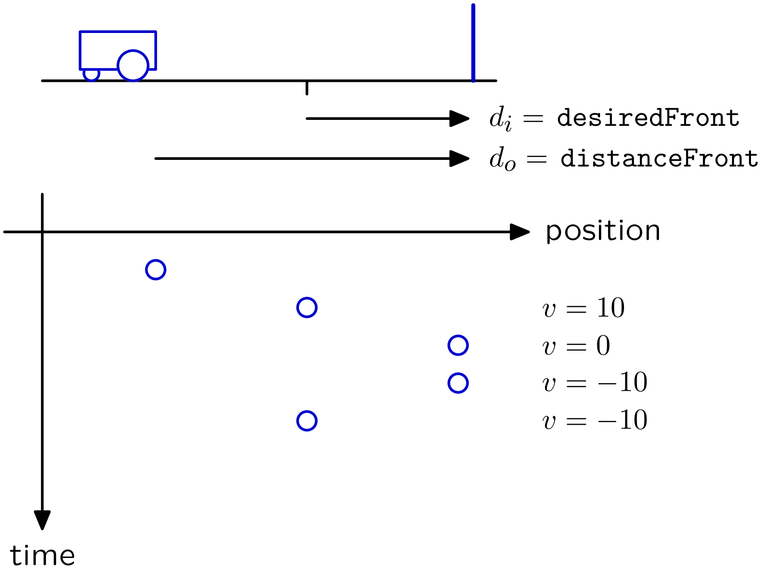

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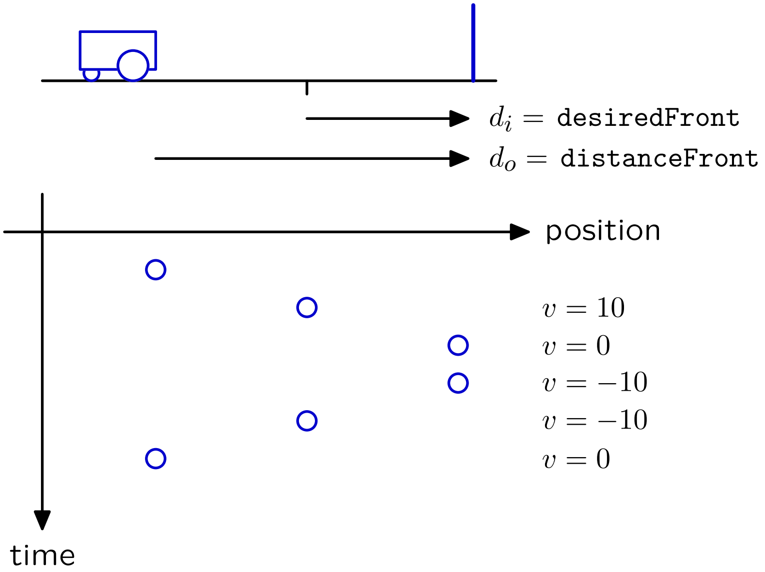
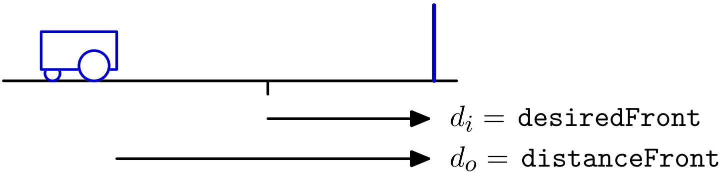


Analysis of wallFinder System: Block Diagram

Incorporating sensor delay in block diagram.



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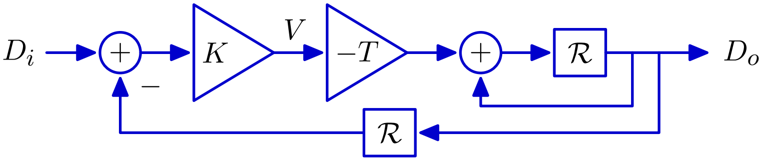
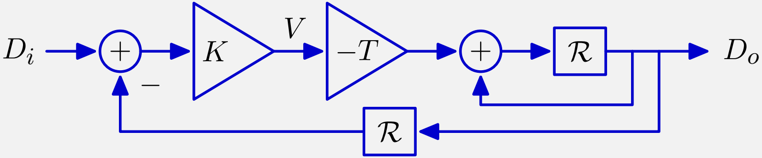


Check Yourself

Find the system function $H = \dfrac{D_o}{D_i}$.



1. $\dfrac{KT\mathcal{R}}{1-\mathcal{R}}$

2. $\dfrac{-KT\mathcal{R}}{1+\mathcal{R}-KT\mathcal{R}^2}$

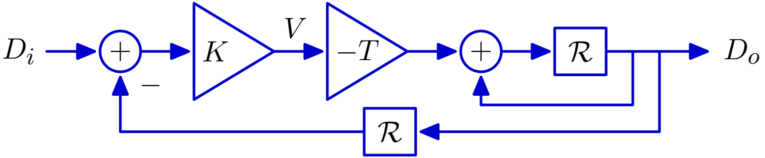
3. $\dfrac{KT\mathcal{R}}{1-\mathcal{R}} - KT\mathcal{R}$

4. $\dfrac{-KT\mathcal{R}}{1-\mathcal{R}-KT\mathcal{R}^2}$

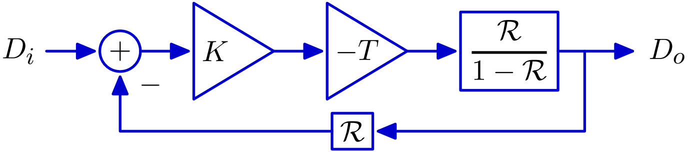
5. none of the above

Check Yourself

Find the system function $H = \frac{D_o}{D_i}$.



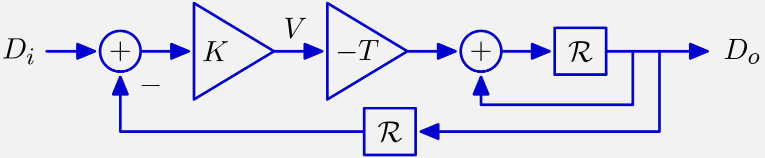
Replace accumulator with equivalent block diagram.



$$\frac{D_o}{D_i} = \frac{\frac{-KT\mathcal{R}}{1-\mathcal{R}}}{1+\frac{-KT\mathcal{R}^2}{1-\mathcal{R}}} = \frac{-KT\mathcal{R}}{1-\mathcal{R}-KT\mathcal{R}^2}$$

Check Yourself

Find the system function $H = \dfrac{D_o}{D_i}$.



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2. $\dfrac{-KT\mathcal{R}}{1+\mathcal{R}-KT\mathcal{R}^2}$
3. $\dfrac{KT\mathcal{R}}{1-\mathcal{R}} - KT\mathcal{R}$
4. $\dfrac{-KT\mathcal{R}}{1-\mathcal{R}-KT\mathcal{R}^2}$
5. none of the above

Analyzing wallFinder: Poles

Substitute $\mathcal{R} \rightarrow \dfrac{1}{z}$ in the system functional to find the poles.

$$\frac{D_o}{D_i} = \frac{-KT\mathcal{R}}{1 - \mathcal{R} - KT\mathcal{R}^2} = \frac{-KT\frac{1}{z}}{1 - \frac{1}{z} - KT\frac{1}{z^2}} = \frac{-KTz}{z^2 - z - KT}$$

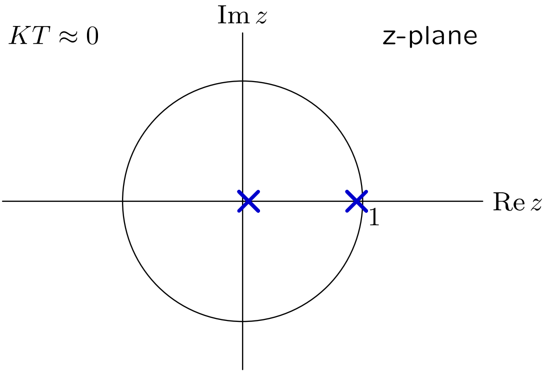
The poles are then the roots of the denominator.

$$z = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + KT}$$

Feedback and Control: Poles

If KT is small, the poles are at $z \approx -KT$ and $z \approx 1 + KT$.

$$z = \frac{1}{2} \pm \sqrt{\ \frac{1}{2}\ ^2 + KT} \approx \frac{1}{2} \pm \sqrt{\ \frac{1}{2} + KT\ ^2} = 1 + KT, -KT$$



Pole near 0 generates fast response.

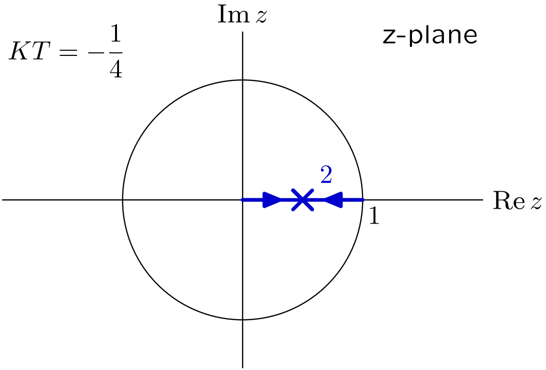
Pole near 1 generates slow response.

Slow mode (pole near 1) dominates the response.

Feedback and Control: Poles

As KT becomes more negative, the poles move toward each other and collide at $z = \frac{1}{2}$ when $KT = -\frac{1}{4}$.

$$z = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + KT} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - \frac{1}{4}} = \frac{1}{2}, \frac{1}{2}$$

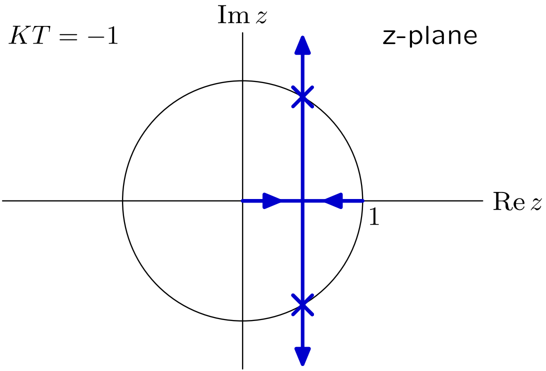


Persistent responses decay. The system is stable.

Feedback and Control: Poles

If $KT < -1/4$, the poles are complex.

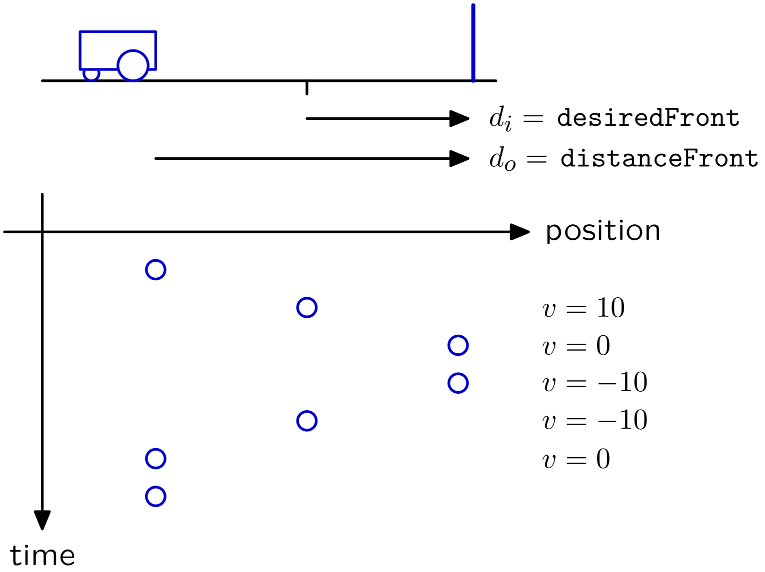
$$z = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + KT} = \frac{1}{2} \pm j\sqrt{-KT - \left(\frac{1}{2}\right)^2}$$



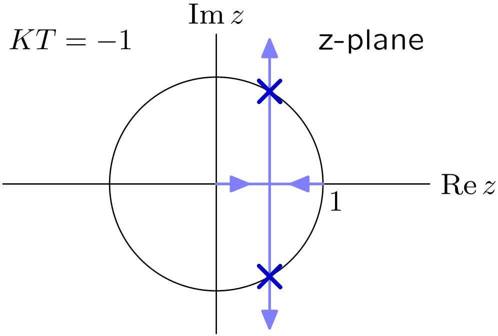
Complex poles → oscillations.

Same oscillation we saw earlier!

Adding delay tends to destabilize control systems.



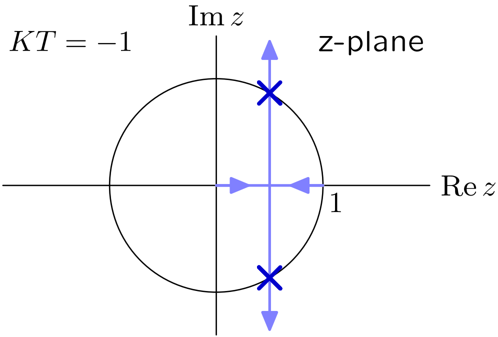
Check Yourself



What is the period of the oscillation?

1. 1
2. 2
3. 3
4. 4
5. 6
0. none of above

Check Yourself

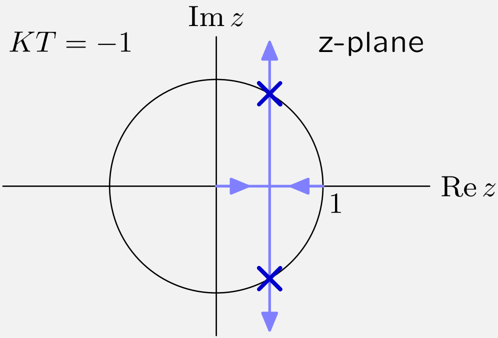


$$p_0 = \frac{1}{2} \pm j\frac{\sqrt{3}}{2} = e^{\pm j\pi/3}$$

$$p_0^n = e^{\pm j\pi n/3}$$

$$\underbrace{e^{\pm j0\pi/3}}_{1},\ e^{\pm j\pi/3},\ e^{\pm j2\pi/3},\ e^{\pm j3\pi/3},\ e^{\pm j4\pi/3},\ e^{\pm j5\pi/3},\ \underbrace{e^{\pm j6\pi/3}}_{e^{\pm j2\pi}=1}$$

Check Yourself

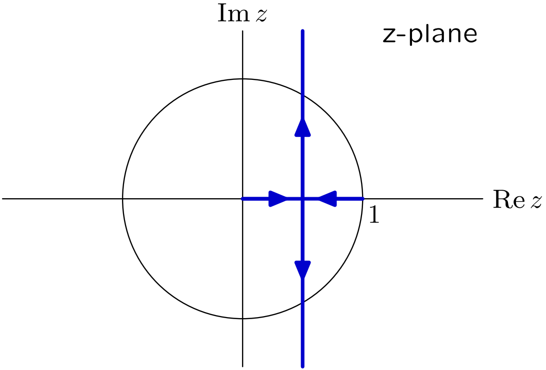


What is the period of the oscillation?

1. 1
2. 2
3. 3
4. 4
5. 6
0. none of above

Feedback and Control: Poles

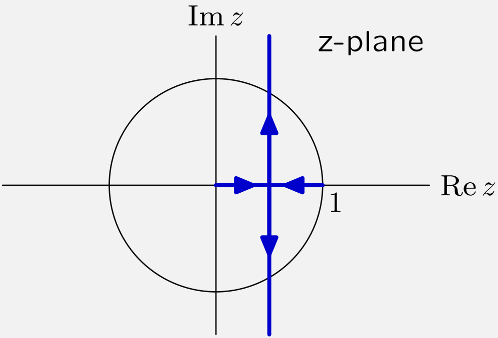
The closed loop poles depend on the gain.



If $KT : 0 \to -\infty$: then $z_1, z_2 : 0, 1 \to \frac{1}{2}, \frac{1}{2} \to \frac{1}{2} \pm j\infty$

Check Yourself

Find KT for fastest response.



closed-loop poles

$$\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + KT}$$

1. 0
2. $-\frac{1}{4}$
3. $-\frac{1}{2}$
4. -1
5. $-\infty$
0. none of above

Check Yourself

$$z = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + KT}$$

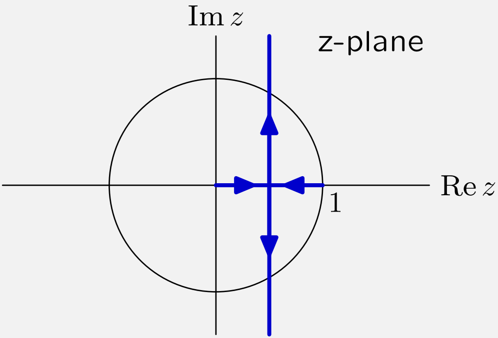
The dominant pole always has a magnitude that is $\geq \frac{1}{2}$.

It is smallest when there is a double pole at $z = \frac{1}{2}$.

Therefore, $KT = -\frac{1}{4}$.

Check Yourself

Find KT for fastest response.



closed-loop poles

$$\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + KT}$$

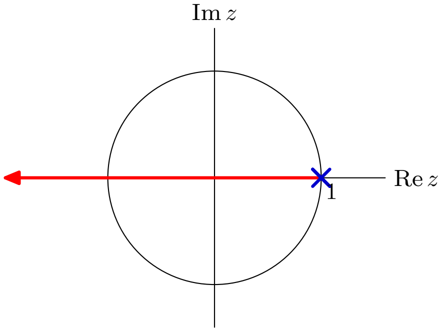
1. 0
2. $-\frac{1}{4}$
3. $-\frac{1}{2}$
4. -1
5. $-\infty$
0. none of above

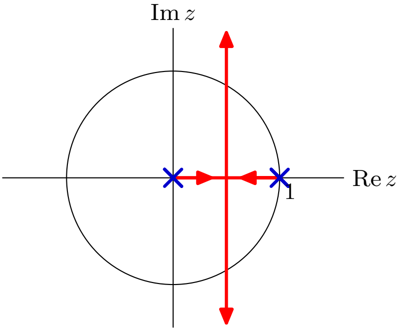
Destabilizing Effect of Delay

Adding delay in the feedback loop makes it more difficult to stabilize.

Ideal sensor: $d_s[n] = d_o[n]$

More realistic sensor (with delay): $d_s[n] = d_o[n-1]$





Fastest response without delay: single pole at $z = 0$.

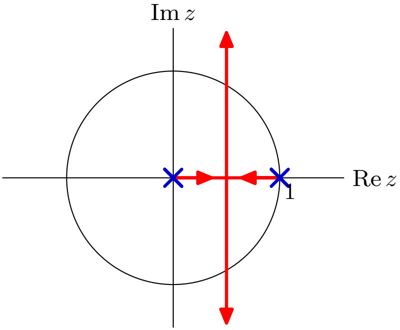
Fastest response with delay: double pole at $z = \frac{1}{2}$. **much slower!**

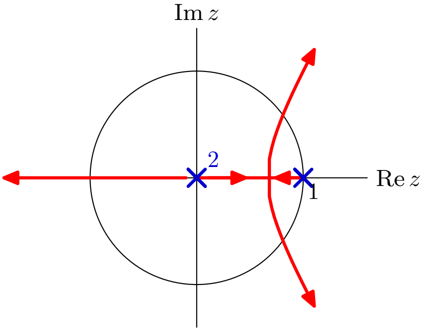
Destabilizing Effect of Delay

Adding more delay in the feedback loop is even worse.

More realistic sensor (with delay): $d_s[n] = d_o[n-1]$

Even more delay: $d_s[n] = d_o[n-2]$





Fastest response with delay: double pole at $z = \frac{1}{2}$.

Fastest response with more delay: double pole at $z = 0.682$.

$\rightarrow$ **even slower**

Feedback and Control: Summary

Feedback is an elegant way to design a control system.

Stability of a feedback system is determined by its dominant pole.

Delays tend to decrease the stability of a feedback system.

Photo from Naval Historical Center Aircraft Data Series.

Block diagram of the F-14 control system as modeled in Simulink® removed due to copyright restrictions. Please see "F-14 Longitudinal Flight Control." The MathWorks, Inc.

6.003 Signals and Systems
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