-- will try it again at the end of this lecture and you show you that stuff hopefully next time. For today we are going to start with nonlinear analysis. Before we do that I wanted to do a little bit of review. I wanted to give you the past three weeks in perspective and show you how all of these things fit into the grand scheme of things. We began by building a great little playground, and within that playground we said that by enforcing upon ourselves the lumped matter discipline we created the lumped circuit abstraction. So within that playfield we assumed that we had dq by dt and d phi by dt to be 0 so that gave us as the lumped circuit abstraction. And within that lumped circuit abstraction, within this playground we looked at several methods of analyzing circuits, including the KVL, KCL method. We also learned the method involving composing resistors, the voltage dividers and so on and solving circuits intuitively. And we also looked at the node method, which is kind of the workhorse of the circuits industry. So when in doubt apply the node method and it will get you where you want to go. Now, we also said that this is good, here is our playground. We said hey, if we focus on those circuits that are linear we come to the left part of our playground. And we said that for linear circuits in this part of the playground we can further use a couple of techniques, a few techniques, superposition, Thevenin, Norton and so on. So these techniques allow you to very quickly analyze complicated circuits, especially when you're looking to find a single current, or voltage or some parameter of interest. Whenever you see, if you see a circuit containing multiple voltage sources or two or more voltage sources or current sources, as a first step think superposition. And so these are very powerful techniques that let you analyze very complicated circuits very effectively. After we did this we said, oh, let me draw another playground here. This is another piece of our playground. And if these are linear circuit then this half of the playground is nonlinear circuits. And we said that if you further focus on discretized values, if you discretized values and focused only on circuits that dealt with binary signals, highs and lows, then we came into this small regime of the playground. And notice that digital circuits are, by their very nature, nonlinear. Remember the circuit, A, B, this was one of our NOR gate circuits? And if you look at transfer functions, that is if I plot, let's say for example, for some combination of input values. Let's say I plot v in verses v out. Let's say, for example, I turned this guy off by setting B to 0 and then I simply apply a low to high transition at v in, then what I would see at the output is a transfer function of the following sort where as v in changes the output switches at some point and then stays at a low value. So when v in is low v out is high and v in and high v out is low. So that's kind of the v out versus v in when B is set at 0. So notice that this is a nonlinear curve. This is not a straight line. It's a nonlinear curve. And so therefore in the digital domain we see highly nonlinear functions that look like this and so on. However, take a look at this circuit. Suppose I focus on the circuit for a given set of switch settings. Let's say, for example, I focus on the circuit when A and B are both 1s. For a given set of switch settings, notice that I'm going to be either in this region or in this region. Notice that this region is a straight line. So if I focus on let's say both A and B at once then I get something like this. And in this situation, for a given set of switch settings, notice that my digital circuit now can be analyzed using linear techniques. So therefore my digital gets moved into the linear domain for a given set of switch settings. So if I fix my switch settings and look at the circuit then each circuit, for a given set of switch settings, is comprised of voltage sources and some resistors and it's a linear circuit. Again, I can go back and apply all my linear techniques to virtually all the digital circuits that you will be dealing with in 6.002. Again, remember if I fix my switch settings, if I fix the inputs then the output can be determined using linear techniques. Because the digital circuits we're showing you in 6.002 simply comprise linear elements like voltage sources and resistors and so on. You'll see some more later. But you can apply your linear techniques and analyze them. The cool thing here is that with just two weeks of stuff that you've learned in 6.002, you are well on our way to being able to analyze certain classes of digital circuits for a given set of switch settings and many, many, many linear circuits. What we will do today is focus on nonlinear circuits. So we look at this space. Notice again that up until now we've dealt with these three methods, which apply to all circuits within this playground, the lumped circuit playground. And the subset of that is the linear domain. And we can analyze linear circuits in this way. And digital circuits, for a given set of switch settings, also fall within this category. So notice that you can go ahead and analyze the digital circuits using superposition or other techniques like that. The next big step for us is to begin our analysis of nonlinear circuits today. The important thing to remember is that nonlinear circuits are also within this big playground in which we are going under the lumped matter discipline. So nonlinear circuits are also lumped circuits. And therefore because we are in that playground we can use any one of our techniques, KVL, KCL or the node method to analyze nonlinear circuits. So if you see a nonlinear circuit, don't get daunted. Just remember this is meant to be simple stuff. So let me simply write down the node equation and analyze it. There is really nothing new in today's lecture. I'm just going to show you a nonlinear circuit and analyzing using techniques that you already know. Today nonlinear circuits. And we look at several methods of analyzing nonlinear circuits. We look at the "Analytic Method". We look at a "Graphical Method". You will look at a "Piecewise Linear Method" in the book. I won't be covering this in lecture. You can read Section 4.4 for the piecewise linear method. In this method you take your curves and you approximate them with a bunch of straight line segments, kind of like the v out, v in curve I've shown you there, and analyze the circuit using linear techniques within any given straight line segment. We will also do incremental analysis. This is also called small signal analysis. So I will cover these two today, I will introduce this one today, and wrap that up during the next lecture. Let's start with a simple example. So I have some voltage, V, some voltage source V. And I have some resistor, R. And I have a fictitious device here that I labeled D. Let's call this fictitious device the "Expo Dweeb". I purposely chose a funky name because this is a fictitious device. Let's call it the Expo Dweeb. And let me write down the associated variables for this device as follows. iD is the current flowing into this terminal and vD is the voltage across this device. So this is a nonlinear device. And this device is characterized by the following equation. Much like resistors were characterized by an iV relation, V is equal to iR, or i is equal to V/R. This device is also characterized by the following element relationship. It's a e raised to bvD. So there is an exponentiation here. Again, this is a fictitious device. And I'll show some funky things that it does in a second. It's a very simple relation. It's an exponential relation where the current relates to the exponentiated value of the voltage vD across the element. So I can plot iD versus vD for this element as follows. Notice that when vD is 0 iD is a, so I have a here, and it looks like this. It's a funny device, a fictitious device. So when vD is 0, I have some current flowing the device, and as vD increases I get an exponential increase in the current through that device. This device is funny in the sense that it is not a passive device in that notice that when vD and iD are positive the product is positive, which is fine, which says that it is consuming power. On the other hand, on the left-hand side notice that the vI relation is negative, which means that when I put a negative voltage on it, it can still sustain a positive current. This must imply that the device is producing power. But for the purpose of a nonlinear analysis we don't have to worry about that. Let's just do it mathematically and find out what it looks like. So back to this again. I have a voltage source, a resistor and my Expo Dweeb connected in that manner. Now, again, reflect on this pattern. A voltage source or a current source, a resistor and some device. This is a very standard pattern you will see again and again and again. In particular, if you look at this device, it's a nonlinear device here. And facing the nonlinear device is a voltage source in series with a resistor. And the reason I say that this is an incredibly important pairing is the following. Notice that if on the left-hand side I had any linear circuit and I had a single nonlinear element in that circuit. Notice that by a Thevenin reduction that you've learned you can take this entire mess. If all you care about is the behavior of the nonlinear device, for the purpose of analyzing this nonlinear device, you can take this entire linear circuit, no matter how complicated it is, voltage sources, current sources, resistors and a bunch of other funky stuff, you can boil all of that down to a Thevenin equivalent, a voltage and a resistor in series. So we can trick you. We can give you a complicated circuit and say ah-ha, tell me what the current is through this device if I apply some voltage, 3 volts there. What you can do is you can say ah-ha, I don't care what happens here so I'm just going to replace the whole thing with a Thevenin equivalent. And you've done your homework now and you can calculate Thevenin equivalents for circuits. And simply replace this and then go ahead and solve the circuit. Again, remember we are engineers. We are looking for answers. We are looking to build interesting systems. And, in general, we like to take the simplest path possible to the solution. So simplify your lives and create a simple Thevenin coupled to a nonlinear device and then you will be rolling. When we talk about a variety of other circuits, nonlinear circuits, time-varying circuits and so on in the rest of this course, we will look at this pattern again and again and again and again until we are blue in the face. And, just remember, the reason we keep looking at this pattern is that whenever you have some big linear mess connected to some interesting device what you can do is if all you care about is analyzing the behavior of that device, you can take this linear mess and simply figure out the Thevenin equivalent, or the Norton equivalent if you like and replace this whole thing with its equivalent and then go ahead and analyze it. So boil an arbitrarily circuit down to a very simple pattern of this sort. What this means is because of this brilliant Thevenin simplification, going forward through the rest of this course we will mostly deal with very simple circuits like this, voltage source, resistor and the device. That's it. Very, very, very rarely will you see multiple sources and lots of resistors in a circuit. It's usually going to be simple stuff. And remember how we got here, by making a Thevenin simplification of a linear mess. All right. If in homeworks or quizzes or in real life, or in many examples of real life, if you find that you have to deal with a lot of grunge and a lot of mess, step back and think a little bit. Try to use intuition and see if you can simplify things using some clever trick or method. Method 1 of analysis. Let's go ahead and analyze this pattern here, this template circuit, if you will, a voltage source a resistor and a nonlinear device. This is the analytical method. And remember the node method applies, so let me go ahead and apply the node method. To apply the node method, what do I do? I first have to select a ground node. Let me insulate this as my ground node. Let me label all the nodes with their voltages. So this node has voltage V and this node has label the capital D. So let me go ahead and analyze this using the node method. So the node method says for each of the nodes in the circuit whose voltage is not known go ahead and write down KCL implicitly applying the element relationships to replace the current values with the voltage values. Let's start with the current going in that direction. Current going from the vD node through resistor R, which looks as follows, vD - V divided by R. That's a current going that way. And the current going down is iD. In general, when I apply the node method, I don't write iD here but I go ahead and write the element relation ae to the bvD here. Then I get an equation in vD and I just solve the mode voltage. However, just to make a couple of extra points later, let me go ahead and do that in two steps, write down this and then go ahead and write down iD separately as ae to the bvD. Again, remember, don't get confused here. In a node method, I don't write down a second step. I directly write down ae to bvD in place of iD. I get one equation in vD, I go solve it. Just for fun today, I'm taking two steps here, writing iD and explicitly putting down iD as ae to the bvD. Now, that's it. I mean this is all there is to it. You guys can now go ahead and analyze nonlinear circuits. You get a bunch of equations, a bunch of unknowns, go solve. I have two equations here. vD and iD are my unknowns and I can just go ahead and solve for them. Now, in general with nonlinear circuits, often times it's hard to get a closed form solution so you may have to use a bunch of methods. You can try a closed form solution or you can try numerical solutions or you can do trial and error. In this case, I'll just go ahead and tell you. Suppose I choose V as 1 volt, R is 1 ohm and b is 1 over volt and a is Âº amps for those values, approximately vD is roughly 0.5 volts and iD is roughly 0.4 volts. You can do this by using trial and error or other methods. In 6.002 we don't dwell on working too hard to solve equations of this sort. If you cannot substitute this in here and solve it directly, we don't ask you to go and learn numerical method and the techniques and so on to solve it. But just remember that you can use trial and error or you can use back substitution and other techniques that you will learn in future numerical methods classes and apply it here. But suffice it to say that, for here we can stick with trial and error if you like. And for these values, vD and iD are 0.5 and approximately 0.4. You're done. It's really that simple. Yes. Oh, I'm sorry. Good catch. I know there is one person that's not sleeping here. Good. So, as I said, there's not a whole lot to it. Whether it's a nonlinear circuit or a linear circuit and as long as I am inside this playground here where the lumped circuit abstraction holds, I can apply my node equations and then go ahead and solve it. Let me show you a few more methods so we can articulate your repertoire of tools for nonlinear circuits. And I'd like to show you a graphical technique. I personally rarely use a graphical technique to solve circuits. And why am I sharing this with you? It turns out that often times by looking at things graphically you can get some better insights into circuit behavior. You can also show cool demos when you show graphs of responses kind of playing with each other and so on. So this is fun for getting intuition and things like that. Graphically all I'm really going to do is solve those two equations graphically. So I'm going to plot equation one. Let me rewrite equation one as follows. iD is -- I'm just rewriting equation one as follows. V/R - vD/R. And I can also draw the second guy -- OK, I can do this as well. I can do an iD versus vD plot. And in this particular situation, you've seen this already, that's my iD versus vD curve right there. And I can do the same for this one here. So this equation establishes the following straight line relationship. It says that when vD is 0, iD is V/R. So that's here. And similarly when iD is 0 then vD is equal to V so I get something here. So that's my straight line relationship corresponding to this equation here. So what I can do is I can simply solve these by superimposing the two curves on the same vD, iD template here and finding the intersection of the curves. So I can take this curve corresponding to two and I can take this curve corresponding to one, and this is V/R and this is V, 0, and I can find the intersection point. This curve here, for reasons that will be obvious about three weeks from now, is called the load line. It's called the load line. You will understand why that is so in a later lecture. So I've given you a template on Page 6 to boil these two down into one equation. So there, again, you can substitute the values for V is 1 volt and R is 1 and so on and so forth and get the same kind of result as you did previously. So there is really nothing new here. All I've done in the second method is combined the two equations graphically and found the solution by looking at where the two curves intersect. At the start of the lecture I also told you that you may want to go and check out the piecewise linear technique -- -- in Section 4.4 of the course notes. All right. For today let me do a third method called "Incremental Analysis". This technique is also called the small signal method. I'm going to show you, before I go into the method, in today's lecture what I'll do is I'll give you a motivating example for why we need the small signal approach. I'll give you a motivating example and show you a little demo. And then I will close with showing you a problem with applying a standard approach, and I'll ask you to see if you can figure out a way to handle it in time for next lecture. So let me give you the motivation here. So here is what I want to do. Many of you have seen one of those electric eye garage door openers, right? You have a receiver at one end and you have some kind of a light beam at the other, and when you walk through it stops, or rather it cuts the circuit and stops the door from closing. And when no one is going through it maintains a connection and lets the door close. So what we did is we went to Home Depot, or one of those stores, and bought a very standard device that essentially produces some response when light impinges on it. And my goal will be to see if I can send music over the light beam using a simple garage door opener device. So here is the little circuit that I will do. We actually went there and built this. I will also show you a demo. Here is my time-varying voltage, vI(t), and this is some music signal. And get some music signal. And I want to connect this to this device, which is a device found in garage door openers. I am going to call it a LED. If you like, you can view it as, this is very similar to our Expo Dweeb. This is called a "Light Emitting Expo Dweeb". That's why it is LED. So what the LED does is, as I apply this voltage across it, that same voltage appears across the Light Emitting Expo Dweeb. And there is some current that flows through the device. And for our analysis we will assume that this device virtually has an identical iD characteristic to the Expo Dweeb just that it emits light. So when I pass a current through it, it emits light. And the light intensity is proportional to the current that flows through. So it emits light and light intensity, LD, is proportional to iD. Here is my little light emitting device, which when current flows through it, itproduces light because its intensity is proportional to the current. And what I will do is I will stick in the receiver here. Think of it as a photo resistor or some other device where I am going to connect that in a circuit. I am not going to spend too much time on this side. I'm going to focus on the left-hand side here. And let's say I have some kind of amplifier and speakers and so on and so forth. Suffice it to say that when the light falls on this device PR that iR that goes through here is proportional to the received light intensity. So if the current is proportional to the received light intensity then I amplify that signal in my amplifier and I get the music playing out here. And notice that the following chain of dependences apply. So I have an input music signal VI. That gets converted to some iD. These are all time-varying signals, so VI is a time-varying signal and so is iD. And iD gets converted to light of some intensity LD. This in turn gets attenuated somewhat and is received at the photo resistor. And I get some intensity LR impinging on that device there. And that in turn produces a current iR and then iR is amplified and goes through a speaker and so on and produces sound. Notice that using this chain I've taken a music signal here and I am playing it here. And just imagine that this is your garage door opener device here where the light emitted is being articulated by the voltage signal VI. And received here. So notice that if I cut this, if I stick something in here and block it then I get no response here, but if I take my hand away then I do get some response. But this is fine. This should work. You could try this at home if you'd like. If you have a garage door opener, just stick a little circuit like this and it should simply work. We have a problem, though. The problem is that, as I said, I'm using the Expo Dweeb here, the light emitting Expo Dweeb, and its characteristics are as follows. iD is exponentially related to the voltage vD, so this is nonlinear. And that's a real problem. Because this is nonlinear, I am going to get a distorted output. Let me show you a little wave form, a little graph to show you how the distortion happens and then show you a little demo showing you the distortion. Let me graphically show you the kind of distortion that is happening here, and I will do it by drawing the following graph. So this is the vD, iD curve for our device. And what I'm going to plot for you is if I have a time-varying vD voltage, I just want to see what the time-varying iD current looks like. And a trick to plot that is to take your input voltage like so. And let's say I apply a sinusoid. So I am just taking a time-varying sinusoidal voltage and rotating the plot 90 degrees like so, so I can see where these points correspond to on that curve. So what this says is that at some point here, for example, where vI, at this point and time, vI is here. Notice vI and vD are the same thing because vI is applied across vD. vI directly applies across the device, and so vI equals vD at all time. So this voltage here corresponds to this voltage, it corresponds to this current and then I can find out what the current is for that voltage. By using the same artifice I can plot the output current iD like so. So for this value I get some current here. And so at time T0 I start here. And notice that as this signal moves up here, I can find out the corresponding values of iD by looking at where a straight line intersects here and plotting the values here. I have a nice little graphical animation to show you this. Hopefully, the laptop will work tomorrow and we can check that. I am doing nothing new here. Just showing you a trick to be able to plot vI versus v out relationships, or vI or versus other relationships based on some kind of a transfer function. So what you end up getting is something that looks like this. Why is that? Notice that this curve here corresponds to the signal. As this signal moves from here to here, this point moves from here to here and that corresponds to this iD. When this moves from here to here that corresponds to a point moving from this part of the curve to here, and that looks like so. And then for the whole negative incursion, notice that the whole negative incursion moves here, so for that entire negative incursion I get an output that looks like this. Notice that this device has completely cut off and hammered negative going signals. What it's done is that rather than giving me a nice little negative spike incursion here, or excursion here, what this is doing is that it is taking this excursion and simply slamming it down to this value here. And then again, when I go back up, I get this peak here. So notice that what was a nice little sinusoid out there gets hammered and squished into this funny curve here. What this device is doing is for positive values it tends to produce exponentially greater current so I get boom, high-rising peaks corresponding to these two, and for negative going voltages it simply compresses them to a low positive value here. And that's what I see here corresponding to negative excursion. So notice that what this will do, if I view sound, if I input sound here, and sound has negative going excursions it will simply scrunch them. But more or less let the positive things through. And that is going to give rise to a bunch of distortion in my signal. So I would like to show you a little demo. Actually, we've gone ahead and built a little device like this. We have an honest to goodness little device costing, I don't know, 50 cents or $1 or something, which is a little voltage, it's a device that emits light proportional to the current flowing through it. I have a receiver. And I am going to play some music, and you will listen to the output here. And hopefully you should see a bunch of distortion because of that effect that I showed you. And what I will do is, before we do that, you will see two curves up there. The yellow, I believe is the vI, is the input, and the green, I believe, is a signal proportionate to -- The other way around. Oh, I see. So green is the input. So green, the lower one is the input and the upper one is the distorted output. So we are going to play some sound through it, music through it and you can listen, through a little CD player. So a couple of things. The good news is that it works. However, I doubt that music artists will come to my studio to record if this is the quality of what I produce. Do notice that there are hardly any negative going excursions in that curve up there, right? All the negative ones have been like scrunched up down into a flat line there, and that's the reason I get this distortion. And just to prove to you that I am indeed using a garage door opener device and not faking it here, I am going to just shut the signal off by stopping the light using a piece of paper here. So notice that this device here is the little device that has a light beam going through the center, and I am going to take this piece of paper, can you turn it up? So let's have some fun with this. If I were to put this piece of paper halfway down, I should get half the intensity, right. So my sound should diminish in volume a little bit. Maybe that will work. Let's see if it works. Nothing to do with 002 but it's just fun. Louder. You can make it loud. Too much coffee. My hand is shaking. I guess you did see the lowering of volume, right? OK. Just way too much coffee, and so my hand was shaking too fast imposing its own sine wave on top of the signal. What did I show you? This was garbage, right? We had a nice little signal input, and the output was completely distorted because I was playing sound over this and this is what happened. Switch to Page 9. Now, this is what I would have liked to have happened. On Page 9 what I would have liked to see happen is this. Suppose I had a light emitting device that looked linear, a straight line where the current was linearly related to vD. Then what I would see, if I had a sinusoid here then I would get a sinusoid here. No distortion there, right? If only things were like I wanted them, if I had a linear device, but I don't have a linear device. I have an Expo Dweeb. Now you know why I call it a dweeb. Well, I'd like a linear device and it's exponential. But this is what I would like. And if I had this I wouldn't show it to you today. If I had this my music would go through without any distortion and I wouldn't have to run cables through my attic. I could just use my garage door opener to play signals from my bedroom and living room and so on, right? So the key thing here is how do I get this? And what I would like you to do is think about it yourselves. What I am given is something like this. This about it yourselves, you know, what would you do? See if you can come to me before lecture tomorrow or Thursday and tell me the answer, OK?