Risk Sharing

Consider H households, with household h consisting of I_h members. There is a single consumption good in this economy. Individuals also care about leisure, thus their per-period utility is $u_{i,h}(c_{i,h}, l_{i,h})$, where i denotes the individual and h denotes the household. This utility function is differentiable in both arguments and satisfies all usual conditions. Individuals discount the future at rate β and maximize expected utility.

Assume that there exist a countable set of payoff relevant states of nature S_t in period t, and we denote a generic state by s_t $(s_t \in S_t)$. As usual we denote a history by $s^t = (s_1, ..., s_t)$. The probability of history s^t is $\pi(s^t)$. The sources of individual income are wage income and non labor income. Non-labor income in history s^t is $y_{i,h}(s_t)$ and the hourly wage rate as $w_{i,h}(s_t)$. Note that both income and wages only depend on the current state s_t. Thus, total wage income of individual i in household h and state s_t will be the wage rate $w_{i,h}(s_t)$ times the number of units of time worked. Each individual has a total time endowment of $T_{i,h}$ per period. Finally, there is a transfer schedule amongst households. Let $\tau_h(s)$ denote the (net) transfer received by household h when state s is realized.

Household Level Analysis

We begin by assuming that the risk sharing unit is the household. Assume a unitarian household model in which allocations are decided as a result of an efficient social-planner-like decision rule with weights $\mu_{i,h}$ on individual utility functions.

(1) Write down the program that a household h solves when deciding consumption and labor allocations for its members.

(2) Characterize the solution to the allocation problem. Please be explicit on which variables $c^{i,h}$ and $l^{i,h}$ depend. Provide a precise intuition why the solution depends on those variables (and why not on some others).

(3) With your answer to the previous question in mind: what do you think of the usual risk sharing regressions? Why might a significant effect of individual income in the consumption regression not be informative about the absence of risk-sharing?

(4) Now suppose that consumption and leisure are separable in individuals' preferences. Formally, suppose that $u^{i,h}(c,l) = v^{i,h}(c) + q^{i,h}(l)$. Which variables determine individual leisure and consumption now?

(5) How could this allocation be decentralized, (assuming that each agent is free to decide how much to work)?

(6) Does individual labor supply of agent i in household h depend on wages and incomes of individuals in the household? Why or why not?

(7) Suppose only for this question that the utility function was CARA only in consumption (i.e.,$u^{i,h}(c) = -e^{-\sigma_{ih}c}$) and you would like to identify the risk aversion of agents in a household. Could you identify the parameter σ_{ih} using the usual risk-sharing regression? Why or why not?

(8) Express the indirect utility function of household h, ω^h, implicitly. What are the arguments of the indirect utility function?

Village Level Analysis

We now assume that the village is the risk sharing unit.

(9) Set up the planning problem for the village and prove that this problem can be solved by the determination of state contingent transfers τ_{ih} to maximize the weighted sum of household indirect utilities.

(10) Characterize the allocation. How would the allocation rule differ in two different states, s and s', that satisfy the following property $\sum_{h=1}^{H}\sum_{i=1}^{I_h} y^{i,h}(s) = \sum_{h=1}^{H}\sum_{i=1}^{I_h} y^{i,h}(s')$?

(11) What regression would you run to test for risk-sharing within the family and across families within the village?

Policy Experiment

Suppose that a new government comes to power and considers that some of the wages paid in the village are extremely low. As a result, the government implements an employment income guarantee scheme in this village (and pays for it using funds from outside). Effectively, this introduces an outside option of w for all individuals in the village and thus all the wages $w^{i,h}(s^t)$ below w become w.

(12) Construct an example for which you can characterize the consumption and leisure allocations across individuals in closed form (or at least in a way that allows you to study how these allocation depend on individual wages). How does consumption and leisure of the "affected" individuals react to the minimum wage? How are the other individuals affected? What drives those results in your example? Do you think those results generalize? [HINT: As I did not tell you what example to pick, there is no "right" or "wrong" in this exercise. Just try to find an example, which works out nicely and discuss your findings.]

14.772 Development Economics: Macroeconomics
Spring 2013