Labor Economics Problem Set 2

A. Intertemporal substitution with no borrowing or lending

1. Use the simple Heckman/MaCurdy utility function $(u\,(c_t, h_t) = c_t^{\gamma} - h_t^{\delta})$ to develop a two-period model, with additively separable preferences over time and goods. Derive the standard λ-constant labor supply specification for this setup.

2. Find an equation defining λ as a function of wages in both periods (assume prices are constant over time at one and that there is no unearned income).

3. Suppose that no borrowing and lending are allowed. Derive the labor supply functions in this case (note: although you can write something down that looks similar, it no longer makes sense to talk about "λ-constant labor supply." Why?)

4. Show that labor supply is more elastic with borrowing and lending than without. Specifically, use this simple model to describe the consequences of a wealth-neutral change in the wage profile with and without access to credit. What does all this have to do with bicycle messengers?

B. CES production

Consider the constant elasticity of substitution (CES) production function

$$F(K, L) = (\alpha K^{\rho} + (1 - \alpha)L^{\rho})^{\frac{1}{\rho}}$$

with $\rho \in (\infty, 1]$ and $\alpha \in (0, 1)$.

1. Derive the conditional factor demands (K^c and L^c) as well as the cost function for the CES production function.

2. The elasticity of substitution is defined as

$$\sigma \equiv -\frac{\partial \log\left(L^c/K^c\right)}{\partial \log\left(w/r\right)}$$

   Derive the elasticity of substitution for the CES production function.

3. The CES production function nests some important special cases.

   (a) What happens to the CES production function when $\rho = 1$? What do we call this production function?

   (b) Compute the limit of the CES production function as $\rho \to 0$. What do we call this production function?

   (c) Compute the limit of the CES production function as $\rho \to -\infty$. What do we call this production function?

C. Fast-food labor demand

Suppose that N small fast food establishments produce according to $f(L) = \log L$, with fixed capital. The produce price is fixed at 1. Aggregate labor supply is given by w^{ϵ} for positive ϵ.

1. Start by assuming firms are price-takers in the factor market (in other wods, the labor market is competitive). Derive an individual firm's demand curve. Use this to derive the aggregate demand curve.

2. Derive the competitive equilibrium wage w_c and aggregate employment level $N \cdot L_c$ as a function of the labor supply elasticity and the number of firms.

3. Now suppose that an evil monopsonist buys the N establishments and operates them as one firm. Assume that the retail fast food market remains competitive. However, the monopsonist has market power in his factor market – in fact he is the sole employer of this type of labor. Derive the new equilibrium wage w_m and the aggregate employment level $N \cdot L_m$ and compare with the competitive equilibrium.

4. Explain how a wise and beneficent government can use the minimum wage to generate the competitive employment level as an equilibrium outcome in spite of the evil monopsonist's market power.

D. Effects of immigration

In recent years, many immigrants have come to Silicon Valley to work in the software industry. Assume that there are two types of programmers: natives (Type 1) and immigrants (Type 2). Initially, assume that there are n_1 native programmers in the labor force and no immigrants. Assume the aggregate labor supply function of native programmers is $n_1 S_1(w_1)$, where $S_1(w_1)$ is the labor supply function of an individual programmer. The demand function for native labor is $D(w)$, with $D'(w) < 0$.

1. Suppose that immigrant programmers have labor supply functions identical to natives' supply functions. Further, assume that firms treat immigrant and native programmers as perfect substitutes. Use graphs to show the effect of an influx of n_2 immigrants on native programmers. Compare this with the effect of immigration if immigrant labor supply is perfectly inelastic.

2. By totally differentiating the conditions for equilibrium in this labor market, find mathematical expressions for the effect of dn_2 on native employment in the two immigrant-elasticity scenarios from part 1. Which effect is larger? What economic parameters does the effect of immigration depend on, and how does it depend on each?
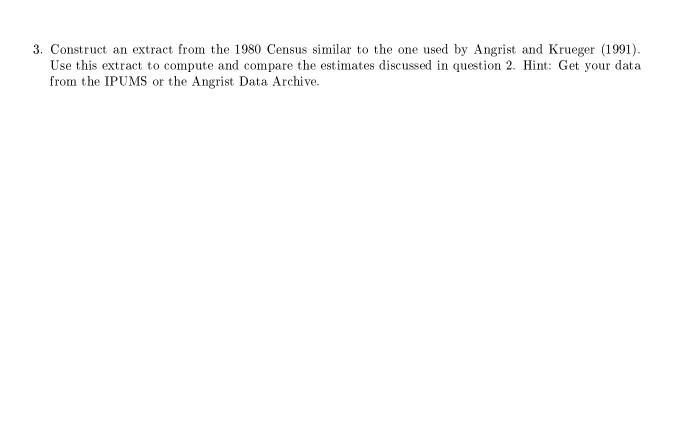
E. IV estimates of earnings functions

1. Consider the following regression equation:

$$y_i = \beta + \rho s_i + \gamma a_i + \varepsilon_i. \tag{1}$$

Assume that the coefficients β, ρ, and γ are defined so that ε_i is uncorrelated with s_i and a_i.

(a) Suppose you estimate a bivariate regression of y_i on s_i instead of (1). Evaluate the plim of the coefficient on s_i in terms of the parameters in equation (1). When does the "short regression" estimate of ρ equal the "long regression" estimate?

(b) Why is the long regression more likely to have a causal interpretation? Or is it?

2. Consider using information on quarter of birth, $Q_i \in \{1, 2, 3, 4\}$, as an instrument for equation (1) when a_i is unobserved. You are trying to use an instrument to get the "long-regression ρ," without observing a_i, using a sample of men born from 1930-39.

(a) Show that using $z_i = 1\,[Q_i = 1]$ plus a constant as an instrument for a bivariate regression of y_i on s_i produces a "Wald estimate" of ρ based on comparisons by quarter of birth. What is the rationale for this estimator? Will it be consistent for ρ in equation (1)?

(b) Suppose that the omitted variable of interest, a_i, is the age of i measured in quarters. Propose two estimation strategies for ρ, one that works when age in quarters is unobserved and one that works when it's observed.

(c) Suppose that all you know is average earnings and average schooling by quarter of birth. Explain how to construct a consistent estimator of ρ. Extract credit: how can you estimate the standard errors from grouped data (means and variances)?

(d) Suppose the equation of interest includes a polynomial function of age in quarters. When and why does identification break down?

3. Construct an extract from the 1980 Census similar to the one used by Angrist and Krueger (1991). Use this extract to compute and compare the estimates discussed in question 2. Hint: Get your data from the IPUMS or the Angrist Data Archive.

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