Lecture Note on Dynamic Insurance

November 2012

- Atkeson-Lucas:
  - basic model of dynamic insurance
  - surprising implication: immiseration
- preferences

$$\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t\theta_t U(c_t) = \sum_{t,\theta^t}\beta^t\theta_t U(c(\theta^t))\,\Pr(\theta^t)$$

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

- $\theta \in \Theta$ finite
- θ_t is i.i.d. with density $p(\theta)$ with $\sum_\theta p(\theta) = 1$; $\Pr(\theta^t) = p(\theta_0)p(\theta_1)\cdots p(\theta_t)$
- rescource constraints

$$\sum_{t,\theta^t} c(\theta^t)\,\Pr(\theta^t) \le e \qquad t = 0, 1, \ldots$$

- First best:
  - FOC

$$\theta_t U'(c(\theta^t)) = \lambda_t$$

and resouce constraint with equality

$$\implies c(\theta^t) = g(\theta_t)$$

where

$$\theta U'(g(\theta)) = \bar{\lambda}$$

- history independent
- consumption rises with θ_t
- not incentive compatible: everyone would report to have the highest shock

- Incentive compatibility:
  - direct mechanism:
    * reports $r_t \in \Theta$, history of reports $r^t \in \Theta^{t+1}$
    * allocation $c(r^t)$
    * strategy $r_t = \sigma_t(\theta^t)$; induces history $r^t = \sigma^t(\theta^t)$
  - truth-telling (IC)

$$\sum_{t,\theta^t} \beta^t \theta_t U(c(\theta^t)) \Pr(\theta^t) \geq \sum_{t,\theta^t} \beta^t \theta_t U(c(\sigma^t(\theta^t))) \Pr(\theta^t) \qquad \forall \sigma$$

- Second best planning problem:

$$\max \sum_{t,\theta^t} \beta^t \theta_t U(c(\theta^t)) \Pr(\theta^t)$$

  subject to IC and RC.

- Approach:
  - study dual
  - relax dual
  - recursive formulation

- Dual:

$$\min e$$

  s.t. IC and

$$\sum_{t,\theta^t} \beta^t \theta_t U(c(\theta^t)) \Pr(\theta^t) = v_0$$

$$\sum_{\theta^t} c(\theta^t) \Pr(\theta^t) \leq e$$

- relaxed dual: replace RC with

$$\sum_{t=0}^{\infty} q^t \sum_{\theta^t} c(\theta^t) \Pr(\theta^t) \leq \sum_{t=0}^{\infty} q^t e$$

  for some $q \in (0,1)$

- Full statement

$$K(v) = \min \sum_{t=0}^{\infty} q^t \sum_{\theta^t} c(\theta^t) \Pr(\theta^t)$$

  s.t. PK

$$\sum_{t,\theta^t} \beta^t \theta_t U(c(\theta^t)) \Pr(\theta^t) = v_0$$

  and IC

$$\sum_{t,\theta^t} \beta^t \theta_t U(c(\theta^t)) \Pr(\theta^t) \geq \sum_{t,\theta^t} \beta^t \theta_t U(c(\sigma^t(\theta^t))) \Pr(\theta^t) \qquad \forall \sigma$$

- utility assignments:

  - chose $u(\theta^t)$ instead of $c(\theta^t)$
  - let $C = U^{-1}$

- Same problem:

$$K(v) = \min \sum_{t=0}^{\infty} q^t \sum_{\theta^t} C(u(\theta^t)) \Pr(\theta^t)$$

$$\sum_{t,\theta^t} \beta^t \theta_t u(\theta^t) \Pr(\theta^t) = v_0$$

$$\sum_{t,\theta^t} \beta^t \theta_t u(\theta^t) \Pr(\theta^t) \geq \sum_{t,\theta^t} \beta^t \theta_t u(\sigma^t(\theta^t))) \Pr(\theta^t) \qquad \forall \sigma$$

- it follows that $K(v)$ is convex and indeed homogeneous:

$$K(v) = A[(1-\sigma)v]^{\frac{1}{1-\sigma}}$$

- recursive version:

  - continuation utility

$$v(\theta^{t-1}) = \mathbb{E}_{t-1} \sum_{\tau=0}^{\infty} \beta^\tau \theta_{t+\tau} u(c_{t+\tau})$$

then

$$v(\theta^{t-1}) = \sum_{\theta_t \in \Theta} [\theta u(c(\theta^{t-1}, \theta_t)) + \beta v(\theta^{t-1}, \theta_t)] p(\theta)$$

- recursive version (drop history notation)

$$v = \sum_{\theta \in \Theta} [\theta u(c(\theta)) + \beta w(\theta)] p(\theta)$$

- temporary incentive constraint

$$\theta u(c(\theta)) + \beta w(\theta) \geq \theta u(c(\theta')) + \beta w(\theta') \qquad \forall \theta, \theta'$$

- Planning problem

$$K(v) = \min \sum_{t=0}^{\infty} q^t \sum_{\theta} [C(u(\theta)) + \beta K(w(\theta))] \Pr(\theta^t)$$

$$v = \sum_{\theta \in \Theta} [\theta u(\theta) + \beta w(\theta)] p(\theta)$$

$$\theta u(\theta) + \beta w(\theta) \geq \theta u(\theta') + \beta w(\theta') \qquad \forall \theta, \theta'$$

- policy functions

$$u(\theta) = g^u(\theta, v)$$
$$w(\theta) = g^w(\theta, v)$$

- homogeneity implies

$$u(\theta) = g^u(\theta, v) = \bar{g}^u(\theta) v$$
$$w(\theta) = g^w(\theta, v) = \bar{g}^w(\theta) v$$

- geometric random walk!
- implication: inequality is ever expanding
- consumption

$$C(u(\theta)) = \bar{g}^u(\theta)^{\frac{1}{1-\sigma}} ((1-\sigma)v)^{\frac{1}{1-\sigma}}$$

- average consumption:

$$\left(\sum_{\theta} \bar{g}^u(\theta)^{\frac{1}{1-\sigma}} p(\theta)\right) ((1-\sigma)v)^{\frac{1}{1-\sigma}}$$

- Q: relaxed problem solves original?
- A: Yes. find q such that

$$\mathbb{E}_{t-1} c_t = \mathbb{E}_{t-1} c_{t+1}$$

$$x_{t-1} \equiv ((1-\sigma)v_{t-1})^{\frac{1}{1-\sigma}} = \mathbb{E}_{t-1}\left((1-\sigma)v_t\right)^{\frac{1}{1-\sigma}} = \mathbb{E}_{t-1} x_t$$

this requires

$$((1-\sigma)v)^{\frac{1}{1-\sigma}} = ((1-\sigma)\bar{g}^w(\theta)v)^{\frac{1}{1-\sigma}}$$

$$1 = \bar{g}^w(\theta)^{\frac{1}{1-\sigma}}$$

- immiseration:

$$x_t = \varepsilon_t x_{t-1}$$

$$\mathbb{E}_{t-1}\varepsilon = 1$$

- implication: (Martingale convergence theorem)

$$x_t \to 0 \qquad \text{a.s.}$$

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