TAX INCIDENCE IN MULTI-SECTOR MODELS

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Two-Sector Models: Partial Factor Taxes

The classical Harberger (1962 JPE) analysis of tax incidence takes place in a two-sector economy. Production takes place in two sectors according to constant returns technology. Both sectors employ labor and capital.

Production: $X_1 = F_1(L_1, K_1)$

$X_2 = F_2(L_2, K_2)$

F_1 and F_2 are constant-returns-to-scale production functions.

Demand: $$X_1 = X_1\left(\frac{p_1}{p_2}\right)$$

$$X_2 = X_2\left(\frac{p_1}{p_2}\right)$$

p_1 and p_2 are the producer prices for goods 1 and 2. No excise taxes, so producer prices equal consumer prices.

Factor Supplies: $\bar{L} = L_1 + L_2$

$$\bar{K} = K_1 + K_2 .$$

These equations assume full employment of both factors, and a fixed total supply of both capital and labor. What "run" does this correspond to? Capital and labor can move across sectors, but total amounts are fixed.

Factor Returns: $w = p_1 F_{1L}(L_1, K_1)$

$$w = p_2 F_{2L}(L_2, K_2)$$

$$r = (1 - \tau)\, p_1 F_{1K}(L_1, K_1)$$

$$r = p_2 F_{2K}(L_2, K_2).$$

τ is a partial factor tax on capital in sector 1. The government is assumed either to return the revenue it collects in lump-sum fashion to households, or to spend it in the same way that households would. (When the government's spending patterns are the same as the private sector's, one wonders why there is a government! But when the government's spending pattern differs from that of the private sector, there can be effects on factor rewards simply from these demand effects.)

Choosing a convenient price normalization can simplify the algebra in analyzing two-sector models. An attractive choice is $w \equiv 1$. With this normalization, studying changes in relative factor rewards, $d(r/w)/d\tau$, reduces studying to dr/τ. Analyzing the system is easiest if we take a slightly different approach and distill the system to five equations in five unknowns:

$$(1)\quad p_1 F_1(K_1, L_1) = L_1 + \left(\frac{r}{1-\tau}\right)K_1$$

$$(2)\quad p_2 F_2(\bar{K} - K_1, \bar{L} - L_1) = (L - L_1) + r(\bar{K} - K_1)$$

$$(3)\quad d\log\left(\frac{K_1}{L_1}\right) = -\sigma_1 d\log\left(\frac{r}{1-\tau}\right)$$

$$(4)\quad d\log\left(\frac{(\bar{K} - K_1)}{(\bar{L} - L_1)}\right) = -\sigma_2 d\log r$$

$$(5)\quad x_1\left(\frac{p_1}{p_2}\right) = F_1(K_1, L_1)$$

The first two equations restate the constant returns assumption. Equations (3) and (4) are just definitions of substitution elasticities, taking account of taxes in sector

one to recognize that the pre-tax marginal product of capital is r/(1-τ). Equation (5) is the goods market equilibrium condition. By Walras' Law, when both factor markets and the market for good 1 clear, the market for good 2 also clears.

Now differentiate this system and translate into log derivatives (ˆ denotes a log derivative such as $\hat{r} = d\log r/r$.)

(1′) $\hat{p}_1 = \alpha_1 \hat{r} + \alpha_1 d\tau$

(2′) $\hat{p}_2 = \alpha_2 \hat{r}$

(3′) $\hat{K}_1 - \hat{L}_1 = -\sigma_1(\hat{r} + d\tau)$

(4′) $\lambda_L \hat{L}_1 - \lambda_K \hat{K}_1 = -\sigma_2 \hat{r}$

(5′) $\eta(\hat{p}_1 - \hat{p}_2) = \alpha_1 \hat{K}_1 + (1-\alpha_1)\hat{L}_1$.

The parameter $\alpha_1 = rK_1 / p_1 X_1$ denotes capital's share in sector 1, $\lambda_K = K_1/(\bar{K} - K_1)$ and $\lambda_L = L_1/(\bar{L} - L_1)$ are the ratios of capital and labor used in sectors 1 and 2, respectively, and $\eta = \dfrac{dX_1}{d(p_1/p_2)} \dfrac{p_1/p_2}{X_1}$ is the elasticity of demand for good 1 with respect to the relative price of good 1 in terms of good 2.

In deriving (5′) from (5), recall that we are free to choose the units of measurement for goods 1 and 2. For example, if we can measure the good "coffee" in arbitrary units, and if one pound of coffee costs $5.00, then by selecting the unit of measurement to be a fifth of a pound of coffee, we can set the price of this "good" to unity. Choosing in this way we can set $p_1 = p_2 = 1$ in the initial equilibrium. This means that $\hat{p}_1 = dp_1$.

To solve (1′) – (5′) for $\hat{r}$ as a function of $d\tau$, we start by substituting out $\hat{p}_1$ and $\hat{p}_2$ in (5′) and (4′), and use (3′) to solve for $\hat{K}_1$ in these expressions. This yields:

(5″) $\eta((\alpha_1 - \alpha_2)\hat{r} + \alpha_1 d\tau) = \alpha_1[\hat{L}_1 - \sigma_1\hat{r} - \sigma_1 d\tau] + (1-\alpha_1)\hat{L}_1$

(4″) $\lambda_L\hat{L}_1 = \lambda_K[\hat{L}_1 - \sigma_1\hat{r} - \sigma_1 d\tau] - \sigma_2\hat{r}.$

It is straightforward to solve (5″) for $\hat{L}_1$:

(5‴) $\hat{L}_1 = [\eta(\alpha_1 - \alpha_2) + \alpha_1\sigma_1]\hat{r} + (\eta\alpha_1 + \sigma_1\alpha_1)d\tau.$

Equation (4″) can be rewritten

(4‴) $(\sigma_2 + \sigma_1\lambda_K)\hat{r} = (\lambda_K - \lambda_L)\hat{L}_1 - \lambda_K\sigma_1 d\tau.$

We can then substitute (5‴) into (4‴) and solve to find:

$$(6)\quad \begin{aligned}&\{\sigma_2+\sigma_1\lambda_K-(\lambda_K-\lambda_L)[(\alpha_1-\alpha_2)\eta+\alpha_1\sigma_1]\}\hat{r}=\\&[(\lambda_K-\lambda_L)\alpha_1(\eta+\sigma_1)-\lambda_K\sigma_1]d\tau.\end{aligned}$$

From this equation we can solve for $\hat{r}$ as a function of $d\tau$:

$$(7)\quad \hat{r}=\frac{(\lambda_K-\lambda_L)(\eta+\sigma_1)\alpha_1-\lambda_K\sigma_1}{\sigma_2+\lambda_K\sigma_1-(\lambda_K-\lambda_L)[(\eta+\sigma_1)\alpha_1-\eta\alpha_2]}d\tau$$

This is the general result of the two-sector analysis. All of the well known simplifications and "intuitions" from this model arise as special cases of (7).

The expression multiplying $d\tau$ in (7) cannot be signed a priori. Rather, we need to consider particular cases. To motivate several of these cases, recall that $\lambda_K=K_1/K_2$ and $\lambda_L=L_1/L_2$. When $\lambda_K-\lambda_L>0,\ K_1/K_2-L_1/L_2>0,$ so $K_1/L_1-K_2/L_2>0.$ If sector 1 is more capital intensive than sector 2, then $\lambda_K-\lambda_L>0.$ Such a condition on the relative capital intensities of the two sectors is important because there are two effects at work in the two-sector analysis.

The partial factor tax on capital in sector 1 induces a substitution effect away from capital in sector 1, and an output effect away from demand for good 1 and toward

demand for good 2. Whether the output effect raises or lowers the demand for capital depends on the relative capital intensities of the two sectors, i.e., on $\lambda_K - \lambda_L$.

Case (i): $\sigma_1 = 0$ (No substitution effect):

In this case (7) becomes

$$(8) \quad \hat{r} = \frac{(\lambda_K - \lambda_L)\eta\alpha_1}{\sigma_2 - (\lambda_K - \lambda_L)\eta(\alpha_1 - \alpha_2)} d\tau.$$

If $\lambda_K - \lambda_L > 0$, then since $\eta < 0$, the numerator is negative. Since $\lambda_K - \lambda_L > 0$ implies
$K_1 / L_1 > K_2 / L_2$, which implies $rK_1 / wL_1 > rK_2 / wL_2$
or $\alpha_1 /(1-\alpha_1) > \alpha_2 /(1-\alpha_2)$, we know $\alpha_1 > \alpha_2$.
This means the denominator in (8) is positive. Thus with $\sigma_1 = 0, \lambda_K > \lambda_L$ implies that r declines. Capital bears some of the tax.

When $\lambda_K < \lambda_L$, however, both the denominator and the numerator are positive, so r rises. Capital shifts more than 100% of the tax burden to labor.

When $\lambda_K - \lambda_L = 0$, or $\eta = 0$, then $\hat{r} = 0$. Neither output nor substitution effects operate, so output prices rise and real returns to capital do not decline. Capital and labor

bear the tax in proportion to their shares of national income.

Case (ii): $\eta = 0$ (No output effect.)

In this setting, equation (7) becomes:

$$(9)\quad \hat{r} = \frac{(\lambda_K - \lambda_L)\sigma_1\alpha_1 - \lambda_K\sigma_1}{\sigma_2 + \lambda_K\sigma_1 - (\lambda_K - \lambda_L)\sigma_1\alpha_1}\, d\tau.$$

This can be rewritten as

$$(9')\quad \hat{r} = \frac{-[\lambda_K\sigma_1(1-\alpha_1) + \lambda_L\sigma_1\alpha_1]\, d\tau}{\sigma_2 + [\lambda_K\sigma_1(1-\alpha_1) + \lambda_L\sigma_1\alpha_1]}.$$

The numerator is negative and the denominator positive, so $\eta = 0$ implies that capital must bear <u>some</u> of the tax.

Case (iii): $\sigma_2 = 0$ (No substitution in sector 2.)

Now, equation (7) becomes:

$$(10)\ \hat{r} = \frac{(\lambda_K - \lambda_L)(\eta + \sigma_1)\alpha_1 - \sigma_1\lambda_K}{\eta\alpha_2(\lambda_K - \lambda_L) - (\lambda_K - \lambda_L)(\eta + \sigma_1)\alpha_1 + \sigma_1\lambda_K}\, d\tau.$$

If $\lambda_K - \lambda_L > 0$, then K_1/L_1 must rise. Further tedious algebra shows that $\hat{r} > -d\tau$. In this case, capital bears more than 100% of tax.

Beyond Two Sectors: Computable General Equilibrium Models

Moving beyond two-sector models makes it difficult to obtain analytic results. Computable general equilibrium (CGE) models were developed to address precisely these concerns. They were pioneered by John Shoven and John Whalley (1972 *Journal of Public Economics*) and have subsequently been a topic of voluminous research in public finance, trade, and development. The most widely used public finance model is described by Charles Ballard, Don Fullerton, John Shoven, and John Whalley in A General Equilibrium Model for Tax Policy Evaluation, (1985). The basic structure of this model is:

Production:

There are 19 different productive sectors with CES production functions for each sector:

(11) $$x_i = \left[K_i^{\beta} + L_i^{\beta}\right]^{\frac{1}{\beta}} \qquad i = 1, ...,19$$

The model incorporates separate tax parameters and production parameters in each sector.

Consumers:

There are 12 different classes of consumers, corresponding to different strata of the income distribution. Each consumer has a CES utility function:

(12) $$u_j = \left[\left(\prod_{i=1}^{19} x_{ij}^{\lambda_i} \right)^{\sigma} + (\overline{H} - L_j)^{\sigma} \right]^{\frac{1}{\sigma}} \qquad j = 1, ..., 12$$

Government:

The government has a Cobb-Douglas utility function. These preferences determine the composition of government spending.

Key Assumptions:

- Uncompensated labor supply elasticity of 0 to 0.15.
- Interest elasticity of saving ranges from 0 and 0.40.
- Elasticities of substitution vary across sectors:

Agriculture	0.61
Food Products	0.71
Clothing	0.82
Paper	0.77
Petroleum Refining	0.74
Vehicle Manufacture	0.82

Here is an illustration of the tax rate calibration for this model. The statistics for capital, labor, and output taxes describe differences across industries (19 sectors):

Capital Taxes: Mean 0.970, Std Dev 0.729

Labor Taxes: Mean 0.101, Std Dev 0.009

Output Taxes: Mean 0.008, Std Dev 0.035

Excise Taxes (15 Goods): Mean 0.067, Std. Dev 0.140

Note that there is substantial heterogeneity across sectors in tax rates in the U.S. in the early 1980s, notably those on capital income, as well as the variation in marginal deadweight burdens from different tax instruments.

Key Findings:

- Marginal deadweight losses are on the order of 35 cents per dollar of revenue for the tax system as a whole.
- Intersectoral variations in tax rates create important distortions.
- Intertemporal distortions are more important than intersectoral ones. This has motivated the great volume of research on the interest elasticity of saving, a key parameter in determining the inter-temporal distortion.

The table below indicates the marginal excess burden from raising extra revenue from specific portions of the tax system:

	Parameter Choices			
Uncompensated Saving Elasticity:	0.0	0.4	0.0	0.4
Uncompensated Labor Supply Elasticity:	0.0	0.0	0.15	0.15
All Taxes	.170	.206	.274	.332
Capital Taxes	.181	.379	.217	.463
Labor Taxes	.121	.112	.234	.230
Consumer Sales Taxes	.256	.251	.384	.388
Sales Taxes on Commodities other than Alcohol, Tobacco, Gasoline	.035	.026	.119	.115
Income Taxes	.163	.179	.282	.314
Output Taxes	.147	.163	.248	.279

Source: Charles Ballard, et.al., 1985, *American Economic Review*

Appendix: Deriving (1′) from (1)

The derivation of equation (1') above is somewhat more complicated than other parts of the derivation. Differentiating (1) yields:

$$X_1 dp_1 + p_1 F_{1K} dK_1 + F_{1L} dL_1 = dL_1 + \left(\frac{r}{1-\tau}\right) dK_1 + \frac{K_1}{1-\tau} dr + \frac{rK_1}{(1-\tau)^2} d\tau. \quad \text{(A1)}$$

Assuming that $\tau = 0$ at the initial point and $w \equiv 1$,

(A2)

$$X_1 dp_1 + \frac{r}{1-\tau} dK_1 + dL_1 = dL_1 + \frac{r}{1-\tau} dK_1 + K_1 dr + rK_1 d\tau$$

which simplifies to

$$dp_1 = \frac{K_1}{X_1} dr + \frac{rK_1}{X_1} d\tau \quad \text{(A3)}$$

or

$$\frac{dp_1}{p_1} = \frac{rK_1}{p_1 X_1} \frac{dr}{r} + \frac{rK_1}{p_1 X_1} d\tau. \quad \text{(A4)}$$

Since $\alpha_1 = rK_1 / p_1 X_1$,this yields equation (1′) above.

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