# Notes on "Bubbles and capital flow volatility: Causes and risk management" by Ricardo Caballero and Arvind Krishnamurthy

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# 1 The basic model — without bubbles

## 1.1 Outline of model

- Domestic economy composed of OLG of risk neutral agents. Consume only when old

- Two types of goods in the economy: domestic goods and international goods. Perfect substitutes in consumption.

- International investors: only care about international goods for consumption

- At each generation, a continuum of measure 1 of agents are born, which are ex-ante identical. All of them are born with an endowment $W_t$ of **international goods,** which can be invested abroad at an interest rate of $r^*$

- When old, $\frac{1}{2}$ of them get an investment opportunity (entrepreneurs) that can turn international goods into domestic goods, and half of them can help finance this projects (bankers).

- Also when old, they have an endowment of **domestic goods**

- Entrepreneurs have limited borrowing capacity

## 1.2 Technology and endowments

At $t$, each agent is born with $W_t$ units of the international good. This can be saved abroad at an interest rate of $r^*$ (can interpret this as a savings technology with rate $r^*$). When old, they receive an endowment of $K_t$ units of domestic goods. Authors simplify (or normalize) it to $K_t = RW_t$ (and $R$ is defined below). Endowment (or population in any case) is assumed to grow at an exogenous growth rate of $g > 0$. Moreover,

for this section, we assume that this is actually the only technology available for agents to save their international goods. If we denote $W_t'$ the amount of international goods available at $t+1$ for agents of generation $t$, we have that

$$W_t' = (1 + r^*) W_t$$

Obviously, we will have that bubbles are possible if and only if $g > r^*$ (present value of endowment of the economy has to be infinite in the equilibrium, following Santos-Woodford) and this is assumed to be the case. This captures the idea that the domestic economy grows faster than the international one.

When old, there's a 50% chance that they will have access to an investment opportunity, that per unit of **international** good $I_t$ gives $RI_t$ units of **domestic** goods ($R > 1$). Since international investors only care about international goods, they would in principle not fund this investment opportunities. This agents are called **entrepreneurs**.

The other half of agents can lend $l_{t+1}$ international goods to entrepreneurs, for them to use them into production and repay $p_{t+1}l_{t+1}$. However, due to limited liability, the value of the loan is constrained to be a fraction of the domestic endowment of entrepreneurs: namely

$$p_{t+1}l_{t+1} \leq \psi R W_t$$

It is moreover assumed that $\psi R < 1$ throughout the paper.

The main goal of the paper is to analyze how young agents save their international goods so they can use them in the investment opportunities. When allowing for stochastic bubbles in the next section, we will see how this changes the savings decisions (which are very restricted in this setting).

## 1.3 Equilibrium

In any equilibrium, we must have that the promise to repay per unit invested ($p_{t+1}$) must satisfy $p_{t+1} \in [1, R]$. The supply of loans is the amount of international goods held by bankers; i.e. $l_{t+1}^S = \frac{1}{2} W_t (1 + r^*)$. Since the investment opportunity gives $R \geq p_{t+1}$ in any equilibrium, then entrepreneurs must hit the constraint: i.e. the aggregate demand for funds is

$$l_{t+1}^D = \frac{1}{2} \frac{\psi R W_t}{p_{t+1}} < \frac{1}{2} W_t < \frac{1}{2} W_t (1 + r^*) = l_{t+1}^S$$

So there are funds held by bankers that won't reach the entrepreneurs. Therefore, in equilibrium we must have that $p_{t+1} = 1$. The authors also point out that there is a net outflow of capital in this economy, even though it has the ability of generating output at a greater rate, and it is

$$\text{Net Outflow} = W_t - (1 + r^*) W_{t-1} = (g - r^*) W_t > 0$$

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## 2   Model with stochastic bubbles

Introduce an asset that bears no dividend (like money), and such that in an equilibrium rate of return is given by the following process:

$$\mathbb{E}\left(\frac{q_{t+1}}{q_t}\right) \equiv 1 + r_{t+1}^b = \begin{cases} 1 + g \text{ with prob. } 1 - \lambda \text{ if } r_t^b = g \\ 1 \text{ with prob. } 1 \text{ if } r_t^b = 0 \\ 0 \text{ with prob. } \lambda \text{ if } r_t^b = g \end{cases} \implies$$

$$r_{t+1}^b = \begin{cases} g \text{ with prob. } 1 - \lambda \text{ if } r_t^b = g \\ 0 \text{ with prob. } 1 \text{ if } r_t^b = 0 \\ -1 \text{ with prob. } \lambda \text{ if } r_t^b = g \end{cases}$$

with $q_t$ the price of the bubbly asset. See that conditional on the bubble not bursting, the return on the bubble is $g > r^*$. however, the expected return is $\widehat{r}^b = g - \lambda (1 + g)$ which implies

$$\mathbb{E}\left(\frac{q_{t+1}}{q_t}\right) = 1 + \widehat{r}^b = (1 - \lambda)(1 + g) < 1 + g$$

If we want agents to invest some of their international goods in the bubble, we need that

$$\widehat{r}^b \geq r^* \iff (1 - \lambda)(g - r^*) - \lambda(1 + r^*) \geq 0$$

This happens when

1. If $\Delta r^b \equiv g - r^*$ is high enough

2. If the probability of bursting is low enough: i.e.

$$(1 - \lambda)(g - r) \geq \lambda(1 + r) \iff \lambda \leq \frac{g - r^*}{1 + g} \tag{1}$$

Let $\alpha_t$ be the share of assets of the portfolio of young agents that corresponds to bubbly assets. Then, the amount of available international funds in the next period is

$$W_t' = (1 - \alpha_t) W_t (1 + r^*) + \alpha_t W_t (1 + r^b) = W_t (1 + r^* + \alpha_t (r^b - r^*)) \equiv W_t'(\alpha_t)$$

### 2.1   Entrepreneurs and Bankers

Suppose that the young agent turns to be an entrepreneur when old. Then, the amount of available assets to invest in her technology is

$$A = \underbrace{RW_t}_{\text{endowment when old}} + \underbrace{RW_t'(\alpha_t)}_{\text{return on portfolio}} + \underbrace{(R - \widetilde{p}_{t+1}) l_{t+1}}_{\text{Net profit from loan}}$$

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where

$$l_{t+1} \leq \frac{\psi R}{\widetilde{p}_{t+1}} W_t$$

using the constraint. If it binds (which does if $\widetilde{p}_{t+1} < R$) then the amount of assets she will held will be $A = RW_t + RW_t'(\alpha) + (R - \widetilde{p}_{t+1}) \frac{\psi R}{\widetilde{p}_{t+1}}$. Note that $\widetilde{p}_{t+1}$ is in fact, a random variable which will take two different values (to be determined in equilibrium): $p_{t+1}^B$ (if the bubble continues) and $p_{t+1}^C$ (if it crashes).

Bankers have international goods $RW_t + \widetilde{p}_{t+1} W_t'(\alpha_t)$

## 2.2 Portfolio Decision

Since the portfolio decisions and equilibria when the bubble already burst are uninteresting (we already did that in the previous section) we focus on paths for which the bubble has not crashed yet. Since with prob. $\frac{1}{2}$ the young agent is a banker when old, then the optimal portfolio choice is the one that maximizes

$$\max_{\alpha \in [0,1]} \mathbb{E} \left( RW_t + W_t'(\alpha) \frac{R + \widetilde{p}_{t+1}}{2} + \frac{R - \widetilde{p}_{t+1}}{2} \frac{\psi R}{\widetilde{p}_{t+1}} W_t \right)$$

which is a linear function of $\alpha$, so it is either a corner solution or an interior solution. For it to be interior (which must happen in equilibrium) we must then have that

$$(1 - \lambda) \frac{\Delta r^b}{1 + r^*} \left( R + p_{t+1}^B \right) = \lambda \left( R + p_{t+1}^C \right) \tag{2}$$

## 2.3 Equilibrium

Suppose that at $t + 1$ the bubble did not crash. Then the amount of assets available to lend by bankers is $W_t' = W_t (1 + r^* + \alpha_t (g - r^*))$. We already know that in equilibrium, the price of loans must be between $1 \leq \widetilde{p}_{t+1} \leq R$, so in any equilibrium we must have $p_{t+1}^B \geq 1$. On the other hand, if we assume that $p_{t+1}^B > 1$ then in order to have market clearing it must be true that every asset is lend to the entrepreneurs, giving

$$l_{t+1} = W_t' \iff W_t' = \frac{\psi R}{p_{t+1}^B} W_t \iff p_{t+1}^B = \frac{\psi R}{1 + r^* + \alpha_t \Delta r^b}$$

therefore, in the equilibrium in which the bubble didn't crash, we must have

$$p_{t+1}^B (\alpha_t) = \max \left\{ 1, \frac{\psi R}{1 + r^* + \alpha_t \Delta r^b} \right\}$$

and note that if $\psi R < 1$ (as we assumed before) this condition reduces to $p_{t+1}^B (\alpha_t) = 1$

Suppose now that the bubble has crashed: then price has to be between 1 and $R$, and moreover, if the constraint binds, then the price should be $\frac{\psi R}{(1 + r^* + \alpha(-1 - r^*))} = \frac{\psi R}{(1 + r^*)(1 - \alpha_t)}$ so in equilibrium:

$$p_{t+1}^C (\alpha_t) = \max \left\{ 1, \min \left\{ \frac{\psi R}{(1 + r^*)(1 - \alpha_t)}, R \right\} \right\}$$

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Then, to get the equilibrium, we must have the equilibrium portfolio decision $\alpha$, which must then satisfy the market clearing condition:

$$(1 - \lambda) \frac{\Delta r^b}{1 + r^*} \left( R + p_{t+1}^B (\alpha) \right) = \lambda \left( R + p_{t+1}^C (\alpha) \right)$$

and we solve for $p_{t+1}^C (\alpha)$ in terms of $\alpha$. The equilibrium determination can be seen in the following picture, where $\alpha^P$ denotes the equilibrium portfolio choice by young entrepreneurs
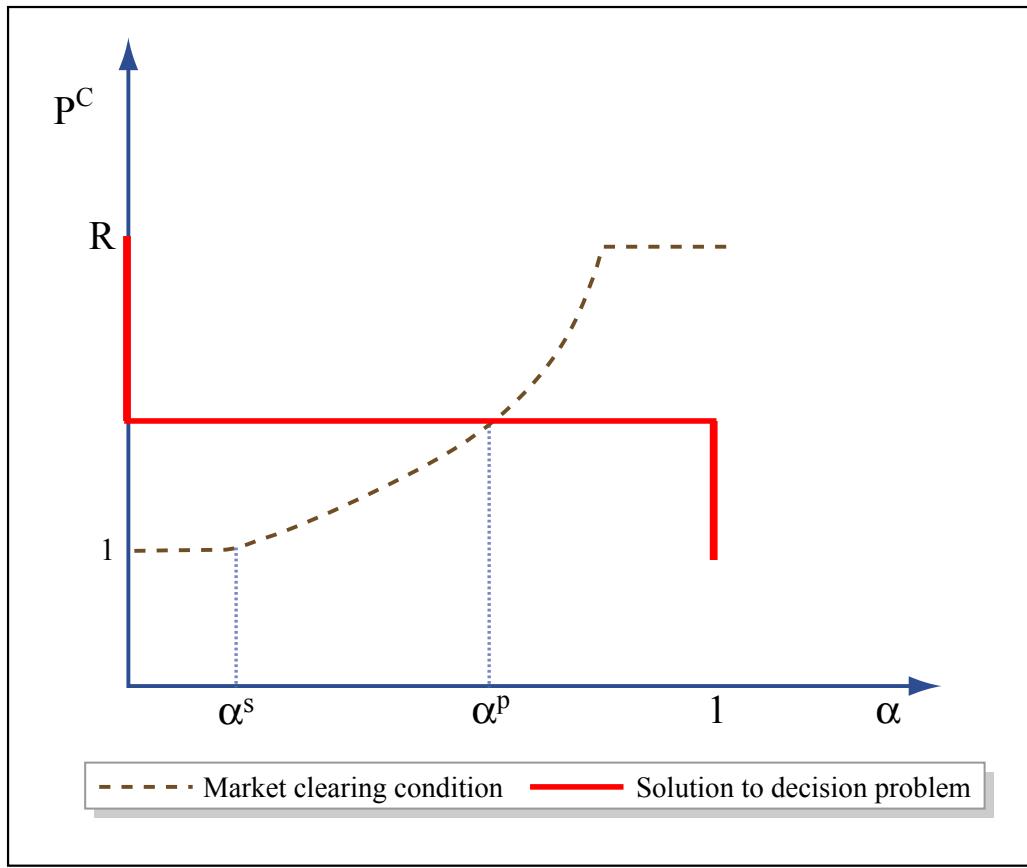


Image by MIT OpenCourseWare.

## 2.4   Welfare effects of portfolio choice

When the bubble crashes, there are two important effects in play

- Direct investment by entrepreneurs in their own projects is limited because they cannot sell their position in the bubbly asset, which reduces aspect

- Indirect investment by banks is also limited, because bankers cannot get funds from young entrepreneurs to sell their assets, which makes them cut the loan supply. Since there are limited commitment constraints, this effect is acentuated, making it possible for interest rates to increase. This is defined by the authors as a "credit crunch" (that is, when the constraints bind)

See from the previous figure that the second effect (the credit crunch effect) is only present when $\alpha > \alpha^S$ (if not, the price of loans is the same as when there is no crash). Authors then show that the welfare maximizing portfolio choice (the one that maximizes expected output, which in this case is exactly expected utility of the representative young agent) is $\alpha^* = \alpha^S$ (that is, the maximum $\alpha$ such that the credit crunch is avoided). The externality comes from the limited liability constraint, which makes the consumption space (in this case, the availability of funds to invest) dependent on the portfolio choice via the price.

Moreover, $\alpha^P$ and $\alpha^S$ depend on $\lambda$ : therefore, the regulation here (which would imply a control over the portfolio choices of agents) depends on the perceived fragility of the bubble.

## 2.5 Ex-ante Desirability of bubbles (not in paper)

The above result on overexposure to risk generated by bubbles crashing is considered fixing the probability of crash to $\lambda$. *We* can show (with some algebra) that the bubble equilibrium is welfare improving if and only if

$$\frac{(1+R)\,\Delta r^b}{\psi R^2 + R\,(1+r^*)} \geq \frac{\lambda}{1-\lambda}$$

that is, if the probability of bursting ($\lambda$) is low enough. We can also show that there exist $\lambda \in [0,1]$ such that all the conditions made on $\lambda$ hold, and such that the previous condition hold, and also the reverse. Therefore, if $\lambda$ is big enough, then bubbles can be ex-ante welfare reducing, absent government intervention.

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