

14.452. Topic 9. Monetary policy. Notes.

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Nr. 1

Look at three issues:

- Time consistency. The inflation bias.
- The trade-off between inflation and activity.
- Implementation and Taylor rules.

Other issues, not touched:

- Disinflation. Optimal speed.
- Liquidity traps. The non negativity constraint for interest rates.
- Exchange rates. Should they matter only through their effect on activity and inflation?

1. Time consistency.

First, a simple case (Kydland-Prescott).

- Phillips curve relation:

$$y_t = \gamma(\pi_t - E_{t-1}\pi_t)$$

Can obviously be rewritten as: $\pi_t = E_{t-1}\pi_t + (1/\gamma)y_t$

- Central bank minimizes :

$$\alpha(y_t - k)^2 + \pi_t^2$$

Cares about output and inflation. Important assumption: $k > 0$. Why?
(First best output level larger than second best)

- Assume the central bank chooses π_t directly.

Time-consistent solution:

At t , taking expectations $E_{t-1}\pi_t$ as given, the central bank minimizes its loss function and chooses:

$$\pi_t = \frac{\alpha\gamma}{1 + \alpha\gamma^2}(\gamma E_{t-1}\pi_t + k)$$

The higher α , or the higher γ , or the higher k , the more attractive inflation. At $t - 1$, people have rational expectations, so $E_{t-1}\pi_t = \pi_t$ and so:

$$y_t = 0, \pi_t = \alpha\gamma k$$

The higher α , the higher γ , the higher k , the higher the inflation rate. Clearly suboptimal. Why? (Wedge and expectations)

If government can commit:

$$y_t = 0, \pi_t = 0$$

How to do it? More on this later.

Time consistency in the NK model

- Let x_t be the (log) output gap, the distance between output and second best (flexible price) output (equivalently, the natural level of output)
- Let the Phillips curve relation be:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1}$$

- Let the central bank minimize

$$E_t \sum_i \beta^i [\alpha (x_{t+i} - k)^2 + \pi_{t+i}^2]$$

Second order approximation to the welfare function. Cares about output and inflation. Again, $k > 0$ because first-best higher than second-best.

- Central bank chooses π_t directly.

Let $\beta^i \mu_{t+i}$ be the Lagrange multiplier. The first order conditions are given by:

$$x_t : \quad -\alpha(x_t - k) - \lambda\mu_t = 0$$

$$\pi_t : \quad -\pi_t + \mu_t = 0$$

$$x_{t+1} \quad E_t[-\alpha(x_{t+1} - k) - \lambda\mu_{t+1}]$$

$$\pi_{t+1} \quad E_t[-\beta\mu_t - \beta\pi_{t+1} + \beta\mu_{t+1}] = 0$$

....

Note the difference between FOC at time t and at time $t+1$ (or any $t+i$, $i > 0$). The presence of lagged μ . Why?

Time consistent policy: Use the current period first-order conditions each period. From the two first-order conditions:

$$x_t = k - (\lambda/\alpha)\pi_t$$

Replace in the Phillips curve to get:

$$\pi_t = \frac{\alpha}{\alpha + \lambda^2} (\beta E_t \pi_{t+1} + \lambda k)$$

Solve forward to get:

$$\pi_t = \frac{\alpha}{\alpha(1 - \beta) + \lambda^2} \lambda k$$

And, by implication, from the Phillips curve, with constant inflation:

$$x_t = \frac{(1 - \beta)\alpha}{(1 - \beta)\alpha + \lambda^2} k$$

If $\beta \sim 1$, $\pi_t \sim (\alpha/\lambda) k$, $x_t \sim 0$ So positive inflation, and zero output gap.
The larger α , or the lower λ , or the larger k , the larger the inflation.

Policy with commitment:

Need to use the set of first order conditions as of t . (could be contingent on shocks, if shocks are present)

Solve for the asymptotic solution, equivalently solution chosen at $t = -\infty$.
(why?)

$$\mu_{t+i} = \mu_{t+i-1} \Rightarrow \pi_{t+i+1} = 0$$

and, from the Phillips curve, zero inflation implies:

$$x_{t+i} = 0$$

How to achieve this?

How to improve on the time-consistent outcome?

- Reputation. Repeated game: If central bank “cheats”, then revert to time-consistent solution forever. (Barro-Gordon)
- Tough central banker (with low α): (Rogoff) Trade-off.
- Increase the cost of inflation (!). (Fischer-Summers.)

So, for example, leave the tax system unindexed (AMT in the US), or not introduce indexed bonds.

May also affect γ however: Indexation of wages.

2. The inflation-output trade-off

Standard wisdom: In response to oil price shocks, central bank should allow for some more inflation to limit the output loss.

Correct? In the basic model, no. Strict inflation targeting is optimal, even with “supply shocks”. “Divine coincidence”.

But the basic model is probably misleading. Distortions and shocks.

Work out the implications of oil price shocks (could do it with technological shocks. do this for variety) (simplified version of Blanchard-Gali)

The essence of the argument in 3 slides. given time constraints...

- In general, three relevant levels of output:

First best (more generally, “constrained efficient”): y_{1t} (in logs)

Second best (defined as the equilibrium without nominal rigidities, also called “natural level”): y_{2t}

Actual (with nominal rigidities): y_t

- Output gap: Distance between actual and second best (natural): $x_t = y_t - y_{2t}$

Welfare relevant output gap: Distance between actual and first best:
 $y_t - y_{1t}$

- In NK model, quite generally a relation between inflation and the output gap:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda(y_t - y_{2t})$$

- Welfare function (second order approximation) a function of the variance of the welfare output gap, and of inflation,

$$V(y_t - y_{1t}), V(\pi_t - \bar{\pi})$$

- Distance first/second best is constant:

$$y_{1t} - y_{2t} = \text{constant}$$

Implications

- Leaving time inconsistency aside, optimal policy is to stabilize $(y_t - y_{1t})$, equivalently stabilize $(y_t - y_{2t})$.
- This also implies stabilizing inflation. No matter what shocks. No policy trade-off. “Divine coincidence.”

Implications

- In the face of an increase in oil prices, keep inflation constant. Output will come down, but this is best. Because second best and first best output also come down.
- Can the result be overturned? Yes, if $(y_{1t} - y_{2t})$ is not constant. Then stabilizing output gap not the same as stabilizing welfare output gap.
- Example: Real wage rigidities as an additional distortion. Then y_{2t} will decline more than y_{1t} . Stabilizing $y_t - y_{1t}$ implies allowing for $(y_t - y_{2t}) > 0$, and some inflation.
- General lesson: Complexity and relevance of welfare analysis. Relevance of distortions.

The detailed model. Assumptions

- Firms: Continuum of monopolistically competitive firms, each producing a differentiated product and facing an isoelastic demand.

The production function for each firm is given by

$$Y = M^\alpha N^{1-\alpha}$$

where Y is output, and M and N are oil and labor used by firm. Focus on shifts in exogenous supply M .

The log marginal product of labor is given by:

$$mpn = (y - n) + \log(1 - \alpha)$$

For future reference, the real marginal cost is given by:

$$mc = w - mpn = w - (y - n) - \log(1 - \alpha)$$

where w is the (log) real wage, taken as given by each firm.

- People: Continuum of households, with utility:

$$U(C, N) = \log(C) - \frac{N^{1+\phi}}{1+\phi}$$

where C is composite consumption (with elasticity of substitution between goods equal to ϵ), N is employment.

The log marginal rate of substitution is given by:

$$mrs = c + \phi n$$

Each good is non-storable. No capital. Consumption of each good must equal output.

Efficient Allocation (First Best)

Assume perfect competition in goods and labor markets. In this case, from the firms' side:

$$w = mpn = (y - n) + \log(1 - \alpha)$$

and, from the consumer-workers' side:

$$w = mrs = y + \phi n$$

where $c = y$. So, first best employment is given by:

$$(1 + \phi) n_1 = \log(1 - \alpha)$$

and, by implication:

$$y_1 = \alpha m + (1 - \alpha) n_1$$

where the index 1 denotes first-best. Employment independent of m . (Why?)

Flexible Price Equilibrium (Second Best)

Take into account monopoly power of firms. From the firms' side, optimal price setting implies $mc + \mu = 0$, where $\mu \equiv \log(\epsilon/(\epsilon - 1))$. So:

$$w = y - n + \log(1 - \alpha) - \mu$$

From the consumer-workers' side:

$$w = mrs = y + \phi n$$

So second best employment and output (also called "natural") are given by:

$$(1 + \phi) n_2 = \log(1 - \alpha) - \mu$$

and, by implication:

$$y_2 = \alpha m + (1 - \alpha) n_2$$

Note:

$$y_1 - y_2 = (\mu(1 - \alpha)/(1 + \phi)) \equiv k$$

Staggered Price Equilibrium

Staggering a la Calvo. So:

$$\pi = \beta E\pi(+1) + \lambda (mc + \mu)$$

where $mc + \mu$ is the log-deviation of real marginal cost from its steady state value, and $\lambda \equiv \delta(1 - \beta(1 - \delta))/(1 - \delta)$, with δ fraction of firms adjusting in any given period.

Can rewrite $mc + \mu$ as:

$$mc + \mu = \left(\frac{1 + \phi}{1 - \alpha} \right) (y - y_2)$$

So:

$$\pi = \beta E\pi(+1) + \gamma (y - y_2)$$

where $\gamma \equiv \lambda(1 + \phi)/(1 - \alpha)$

Inflation depends on the “output gap”, log distance of actual output from natural (second best) output. Relation marginal cost/output gap robust.

The central bank optimization problem

$$\min E_t \sum_i \beta^i [\alpha (y_{t+i} + y_{1,t+i})^2 - \pi_{t+i}^2]$$

subject to:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (y_t - y_{2,t})$$

$$y_{1,t} - y_{2,t} = k$$

- $k > 0$: Time consistency issue. Leave this aside and assume central bank can fully commit.
- Then stabilizing $(y_t - y_{1,t})$ is equivalent to stabilizing $(y_t - y_{2,t})$.
- Optimal policy: $\pi_t = \pi = 0, (y_t - y_{2,t}) = 0$
- “Divine coincidence:” Keeping inflation constant keeps output at its second best (natural) level. No trade-off

Implications

- Even under “supply shocks”, such as an increase in the price of oil (a decrease in the endowment of oil here). Why?
- Output goes down, but so should it. First best also goes down.
- Fairly dramatic result: The central bank can focus just on inflation. It will have the right implications for activity.
- Seems too strong. Even central banks do not follow this principle, and allow for some more inflation for some time.
- Plausible ways out? Something missing from the model?

Way out I. “Cost shocks”

A standard (but unacceptable) way out. Add “cost shocks” to the inflation equation:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (y_t - y_{2,t}) + u_t$$

Then:

- Delivers trade-off between stabilization of inflation and stabilization of the output gap.
- What is u_t ? Not oil price shocks, as we saw they do not appear in the equation (subsumed through their effect on y_2)

- A potential rationale for cost shocks: “Markup shocks”, so $y_{1t} - y_{2t} = k + u_t$. Rewrite inflation equation as:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (y_t - y_{1,t} + k) + \gamma u_t$$

- No longer want to stabilize output at the natural rate. If adverse markup shock, want to allow for more inflation, to keep y_t closer to unchanged $y_{1,t}$.
- What are these markup shocks? Are they important?
- Even in that case, still no inflation accomodation of price of oil shocks. (unless positively correlated with u_t shocks.)

Ways out II. Distortions and shocks

- Suppose distortions interact with shocks, so shocks affect $y_1 - y_2$. Then, typically trade-off.

- One very relevant distortion: Real wage rigidity.

Suppose $m_t = E m + \epsilon_{mt}$. Suppose real wages are set equal to their value if $\epsilon_{mt} = 0$ and do not adjust (extreme case of Blanchard Gali).

- Then y_1 unaffected. $y_{2,t} = y_1 - k + \epsilon_{mt}$. If unexpected decrease in m_t , second best level of output decreases, first best not.
- Replace in Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (y_t - y_1 + k) - \gamma \epsilon_{mt}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (y_t - y_1 + k) - \gamma \epsilon_{mt}$$

- Trade-off. Stabilizing inflation implies $y_t = y_1 - k + \epsilon_{mt}$. Undesirable recessions in response to oil shocks.
- Stabilizing distance from first best implies: $\pi_t = -\gamma \epsilon_{mt}$. Some inflation accomodation to oil shocks.

Conclusions.

- Inflation targeting equivalent to keeping output gap (distance of output from natural level) equal to zero.
- May not be the best policy if shocks affect the distance of second best to first best. Then, trade-off.
- Best policy depends on the nature of distortions.

3. Interest rate rules.

- So far, assumed CB controlled inflation directly. Not the case. Controls high powered money or the short-term interest rate.
- Shift in focus from money growth to interest rate rules. In many models used by CBs, money not present. (Given interest rate, can solve back for money using money demand)
- A very popular rule. Introduced by Taylor as a descriptive device. Known as the “Taylor rule”.

Start from standard NK model:

$$y_t = E_t y_{t+1} - r_{t+1} + \epsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t$$

$$x_t \equiv y_t - y_{2,t}, \quad y_{1,t} - y_{2,t} = \delta$$

- Notation as before. y, y_1, y_2 , actual, first best, second best (natural) log levels of output.
- Unit elasticity of substitution in IS for notational convenience. Shocks to IS; source: Tastes, or government spending.
- Assume no “cost shocks” or “distortions and shocks interactions”.
- Take movements in $y_{2,t}$ as given/unexplained.

- Define $r_{2,t+1}$ as the real interest rate associated with the second best level of output. Called the “natural real interest rate”, associated with “natural output level”. (Wicksell.) Implicitly defined by:

$$y_{2,t} = E_t y_{2,t+1} - r_{2,t+1} + \epsilon_t$$

- Replace, to get two equations in x and π :

$$x_t = E_t x_{t+1} - (r_{t+1} - r_{2,t+1})$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t$$

- Divine coincidence holds, so central bank wants to achieve zero inflation and zero output gap.
- CB has control of the short nominal rate i_{t+1} .

$$r_{t+1} = i_{t+1} - E_t \pi_{t+1}$$

Interest rate rule 1: $i_{t+1} = r_{2,t+1}$

This seems like a plausible rule. Zero inflation and the natural rate of interest.
In this case:

$$x_t = E_t x_{t+1} + E_t \pi_{t+1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t$$

- One solution: $x_t = \pi_t = 0$
- But not the only solution. Look more closely:

Write down the system in matrix form:

$$\begin{vmatrix} x_t \\ \pi_t \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ \gamma & \gamma + \beta \end{vmatrix} \begin{vmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{vmatrix} + \begin{vmatrix} 0 \\ kx_t \end{vmatrix}$$

Roots of the matrix A :

$$\lambda_1 \lambda_2 = \beta, \quad \lambda_1 + \lambda_2 = 1 + \gamma + \beta$$

For uniqueness, the matrix should have both roots inside the circle. If $\gamma \geq 0$, one root is outside. An infinity of converging solutions.

Interpretation: Lack of a nominal anchor.

Interest rule 2: A Taylor rule

Consider the feedback rule: $i_{t+1} = r_{2,t+1} + \phi_\pi \pi_t + \phi_x x_t$

The IS relation becomes:

$$x_t = E_t x_{t+1} - (\phi_\pi \pi_t - \phi_x x_t)$$

Stability and uniqueness iff (Bullard and Mitra 2002):

$$\gamma(\phi_\pi - 1) + (1 - \beta)\phi_x > 0$$

Satisfied if $\phi_\pi > 1, \phi_x = 0$, or $\phi_\pi = 0, \phi_x > 0$

Interpretation:

$$\frac{dr}{d\pi} = \left(\phi_\pi - 1 + \frac{\phi_x(1 - \beta)}{\gamma} \right)$$

Implications and extensions

- Optimal rule? Within the model, choose $\phi_\pi = \infty$. Why not?
- Observability of $r_{2,t+1}$. If not, then how good is a Taylor rule, with \bar{r}_2 replacing $r_{2,t+1}$?
- Robustness to different shocks, lag structures? Gali simulations. 2002.