

14.452. Topic 6. Introducing money

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Nr. 1

1. Motivation

No role for money in the models we have looked at. Implicitly, centralized markets, with an auctioneer:

- Possibly open once, with full set of contingent markets. (Remember, no heterogeneity, no idiosyncratic shocks.) (Arrow Debreu)
- More appealing. Markets open every period.

Spot markets, based on expectations of the future. For example, market for goods, labor, and one-period bonds. A sequence of **temporary equilibria** (Hicks). (no heterogeneity)

Still no need for money. An auctioneer. Some clearing house.

So need to move to an economy where money plays a useful role. The required ingredients:

- No auctioneer. Geographically decentralized trades.
- Then, problem of double coincidence of wants. Barter is not convenient. Money, accepted on one side of each transaction, is much more so.

Two types of questions:

Foundations of monetary theory questions:

- Why money? What kind of money will emerge?
- Can there be competing monies? silver/gold. dollars/domestic currency.
- Fiat versus commodity money?
- Numeraire versus medium of exchange? Should they be the same, or not?

Not just abstract, or history. The rise of barter in Russia in the 1990s. Dollarization in Latin America and hysteresis. “Units of account”, i.e. a numeraire different from the medium of exchange, in Latin America.

But most of the time, we can take it as given that money will be used in transactions, that it will be fiat money, and that the numeraire and the medium of exchange will be the same.

If we take these as given, then we can ask another set of questions:

- How different does a decentralized economy with money look like?
- What determines the demand for money, the equilibrium price level, nominal interest rates?
- How does the presence of money affect the consumption/saving choice?
- Steady state and dynamic effects on real activity and inflation of changes in the rate of money growth.

Start by looking at a benchmark model. Cash in advance (CIA).

Then, look at variations on the model; money in the utility function.

Then focus on price and inflation dynamics, especially hyperinflation.

2. A cash-in-advance model

Ignore uncertainty (need to reintroduce it if look at unexpected changes in money).

Also ignore the labor/leisure choice. This will make the basic structure of the model much easier to understand.

The optimization problem of consumers

$$\sum_{i=0}^{i=\infty} \beta^i U(C_{t+i})$$

subject to:

$$P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + M_t + (1+i_t)B_t + (1+r_t)P_t K_t + X_t$$

and

$$P_t C_t \leq M_t + X_t$$

$$P_t C_t + M_{t+1} + B_{t+1} + P_t K_{t+1} = W_t + \Pi_t + M_t + (1 + i_t) B_t + (1 + r_t) P_t K_t + X_t$$

- P_t is the price of goods in terms of the numeraire (the price level).
- M_t , B_t , K_t are holdings of money, bonds, and capital at the start of period t .
- W_t and Π_t are the nominal wage and nominal profit received by each consumer respectively. i_t is the nominal interest rate (in dollars, not goods) paid by the bonds.
- r_t is the rental rate (in goods) on capital. Money pays no interest.
- X_t is a nominal transfer from the government (which has to be there if and when we think of changes in money as being implemented by distribution of new money to consumers).

$$P_t C_t \leq M_t + X_t$$

- Consumers care only about consumption. Do not derive utility from money.
- The first constraint is the budget constraint in nominal terms.
- If the only constraint was the first, then people would hold no money: Bonds pay interest, capital pays a rent, money does not.

The second constraint explains why people hold money. Known as the **cash in advance** (CIA) constraint. People must enter the period with enough nominal money balances to pay for consumption.

- One potential story: Households: a worker and a consumer. The worker goes to work. The consumer goes to buy goods, before the worker has been paid.
- More sophisticated formulations? For example: The cost of buying consumption goods decreasing in money balances. Return to this below.

Let $\lambda_{t+i}\beta^i$ be associated with the budget constraint (the shadow value of wealth), $\mu_{t+i}\beta^i$ be associated with the CIA constraint (the shadow value of liquidity). Set up the Lagrangian and derive the FOC.

$$C_t : \quad U'(C_t) = (\lambda_t + \mu_t)P_t \quad \Leftrightarrow \quad \left(\frac{1}{P_t}C_t = \lambda_t + \mu_t\right)$$

$$M_{t+1} : \quad \lambda_t = \beta(\lambda_{t+1} + \mu_{t+1})$$

$$B_{t+1} : \quad \lambda_t = \beta(1 + i_{t+1})\lambda_{t+1}$$

$$K_{t+1} : \quad \lambda_t P_t = \beta(1 + r_{t+1}) \lambda_{t+1} P_{t+1}$$

We can manipulate these conditions to get an intertemporal condition on the marginal utility of consumption:

From the third and the fourth:

$$(1 + i_{t+1}) = (1 + r_{t+1}) \frac{P_{t+1}}{P_t} = (1 + r_{t+1})(1 + \pi_{t+1})$$

From the first and second:

$$\lambda_t = \beta \frac{U'(C_{t+1})}{P_{t+1}} \quad \lambda_{t+1} = \beta \frac{U'(C_{t+2})}{P_{t+2}}$$

Replacing in the third:

$$\frac{U'(C_{t+1})}{P_{t+1}} = \beta(1 + i_{t+1}) \frac{U'(C_{t+2})}{P_{t+2}}$$

Divide both sides by $(1 + i_{t+1})$, and then multiply and divide the second term by $(1 + i_{t+2})$, to get:

$$\frac{U'(C_{t+1})}{1 + i_{t+1}} = \beta(1 + r_{t+2}) \frac{U'(C_{t+2})}{1 + i_{t+2}}$$

Interpretation:

- Because people have to hold money one period in advance, the effective price of consumption is not 1 but $1 + i$.
- Once we adjust for this price effect, get the same old relation, between marginal utility this period, marginal utility next period, and the real interest rate.
- Note the role of both the **nominal** and the **real** interest rates.
- Note that if the nominal interest rate is constant, the equation reduces to the standard Euler equation (Why not C_t and C_{t+1} ?)

$$U'(C_{t+1}) = \beta(1 + r_{t+2})U'(C_{t+2})$$

Turn to the characterization of money demand. From the second and third condition:

$$\mu_{t+1} = i_{t+1} \lambda_{t+1}$$

So as long as the nominal interest rate is positive, μ will be positive, and so:

$$\frac{M_t + X_t}{P_t} = C_t$$

Pure quantity theory. No interest rate elasticity.

Equilibrium

- Assume firms rent capital and labor, so

$$P_t F_N(K_t, N_t) = W_t \qquad F_K(K_t, N_t) = r_t - \delta$$

- Given constant returns and competitive markets, pure profit $\Pi_t = 0$
- Employment, N_t is equal to 1 (inelastic supply)
- Money introduced through lump sum transfers to consumers (“helicopter drops”). (Alternative: Money used by the government to buy goods).

$$M_{t+1} - M_t = X_t$$

- As bonds are issued by agents (not firms, renting capital and labor services) and all agents are identical, in equilibrium, there are no bonds outstanding

$$B_{t+1} = B_t = 0$$

Replacing all these conditions in the accumulation equation of consumers-workers gives:

$$P_t C_t + P_t K_{t+1} = P_t F_N(K_t, 1) + (1 + F_K(K_t, 1) - \delta) P_t K_t$$

Or, dividing by P_t , and reorganizing:

$$K_{t+1} = F(K_t, 1) + (1 - \delta)K_t - C_t$$

So, exactly the same condition as in the non monetary economy.

Putting the relevant equations together:

$$\frac{U'(C_{t+1})}{1 + i_{t+1}} = \beta(1 + r_{t+2}) \frac{U'(C_{t+2})}{1 + i_{t+2}}$$

$$(1 + i_t) = (1 + r_t)(1 + \pi_t)$$

$$(1 + r_t) = 1 - \delta + F_K(K_t, 1)$$

$$\frac{M_t + X_t}{P_t} = C_t$$

$$K_{t+1} = F(K_t, 1) + (1 - \delta)K_t - C_t$$

Steady state

Let growth of nominal money be x , so $(X_t/P_t) = x(M_t/P_t)$ From the FOC of the consumers, and the demand for capital by firms:

$$(1 + r) = 1 + F_K(K, 1) - \delta = 1/\beta$$

Same as without money: Modified golden rule.

Using these relations in the budget constraint gives:

$$C = F(K, 1) - \delta K$$

Money growth has no effect on activity. **Superneutrality**

In steady state, real money balances must be constant, so:

$$\pi = x$$

Inflation is equal to money growth. And so, $i = \pi + r = x + r$. **Fisher effect** (from Irving Fisher).

Dynamics

- Harder to characterize. Come from the consumption wedge. $x \rightarrow \pi \rightarrow i$. Easy however to guess the solution to simple paths for money.
- An unexpected permanent increase in money leads to an equal proportional increase in the price level, and no change in the real variables.
- An unexpected permanent increase in money growth, from x to x' . In this case, this leads to a proportional increase in the price level today, and inflation at rate x' from then on. Nominal interest rates increase by $x' - x$. Real variables are unaffected.
- Any path that generates a decrease in the interest rate and an increase in consumption in response to an increase in nominal money growth? (Stylized facts) Not obvious:
Think temporary increase in money growth: temp increase in inflation, in nominal rates. Temporary decrease in consumption.

Conclusions

- We have seen how we can introduce money as a medium of exchange in GE models. The monetary economy does not look very different from the economies we had looked at until now.
- The consumption/saving choice is modified, but not transformed. There is now a demand for money is a function of transactions.
- The real effects of money are limited. In steady state, money is not only neutral, but superneutral
- Changes in money may have dynamic effects on real variables. But these effects appear to be limited.

Extensions

- The CIA model we looked at had an extremely simple cash in advance constraint.
- More general ones, with money demand function of interest rate. GE version of Baumol Tobin, by Romer (See BF, Chapter 4)
- Baumol-Tobin: Households decide how often to go to the bank to exchange money for bonds. The higher the interest rate, the more often they go, the lower their average real money balances.
- In these models, changes in money have distributional effects (between those who go to the bank and those who do not) and thus effects on real activity. But effects appear limited (and exotic).
- CIA models can become very unwieldy. Short cuts? Money in the utility function.

3. Money in the Utility function

Consider the following optimization problem (Sidrauski model):
Consumers/workers maximize:

$$\sum \beta^i U(C_{t+i}, \frac{M_{t+i}}{P_{t+i}})$$

subject to:

$$P_t C_t + M_{t+1} + B_{t+1} = W_t + \Pi_t + M_t + (1 + i_t)B_t + X_t$$

(Consumers/workers own the firms)

Think of the utility function as a reduced form of a more complex problem in which by holding more money, households can shop more efficiently, increase leisure time, and so on.

Plausibly $U_m > 0$ and $U_{mc} \geq 0$ (why?).

First order conditions

For simplicity, ignore uncertainty. Let $\lambda_{t+i}\beta^i$ be the lagrange multiplier associated with the constraint. Then the FOC are given by:

$$C_t : \quad U_c(C_t, \frac{M_t}{P_t}) = \lambda_t P_t$$

$$B_{t+1} : \quad \lambda_t = \lambda_{t+1}\beta(1 + i_{t+1})$$

$$M_{t+1} : \quad \lambda_t = \beta[\lambda_{t+1} + \frac{1}{P_{t+1}}U_m(C_{t+1}, \frac{M_{t+1}}{P_{t+1}})]$$

Rewrite as:

An **intertemporal condition**:

$$U_c(C_t, \frac{M_t}{P_t}) = \beta(1 + r_{t+1})U_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}})$$

An **intratemporal condition**

$$U_m(C_t, \frac{M_t}{P_t})/U_c(C_t, \frac{M_t}{P_t}) = i_t$$

Ratio of marginal utilities has to be equal to the opportunity cost of holding money, so i , the nominal interest rate.

If for example,

$$U(C, M/P) = \log(C) + a \log(M/P)$$

(not an appealing assumption, as $U_{cm} = 0$, but very convenient) Then,

$$(1/C_t) = \beta(1 + r_{t+1})(1/C_{t+1})$$

$$(M_t/P_t) = a(C_t/i_t)$$

This gives us an *IS* and an *LM* relation.

- The IS: the higher the interest rate, the lower consumption given expected consumption in the future.
- The LM: The demand for money is a function of the level of transactions, here measured by consumption, and the opportunity cost of holding money, i .

Under these assumptions, no distortion in the intertemporal consumption decision.

Steady state

The firms' side is the same as before, so:

$$1 + F_K(K, 1) - \delta = 1/\beta$$

$$C = F(K, 1) - \delta K$$

$$U_m(C, \frac{M}{P})/U_c(C, \frac{M}{P}) = (x + r)$$

So, same real allocation again. Superneutrality of money. And a level of real money balances inversely proportional to the rate of inflation, itself equal to the rate of money growth.

What is the optimal rate of money growth?

- As money is costless to produce, the optimal rate is such as to drive the marginal utility of real money to zero, so to drive $i = x + r$ to zero.
- In other words, it is to have $x = -r$: Money growth—or inflation—negative and equal to minus the marginal product of capital.
- This result is known as the **Optimum Quantity of Money** (the semantics are not great, as this is a result about money growth, not level.)

Dynamics and conclusions

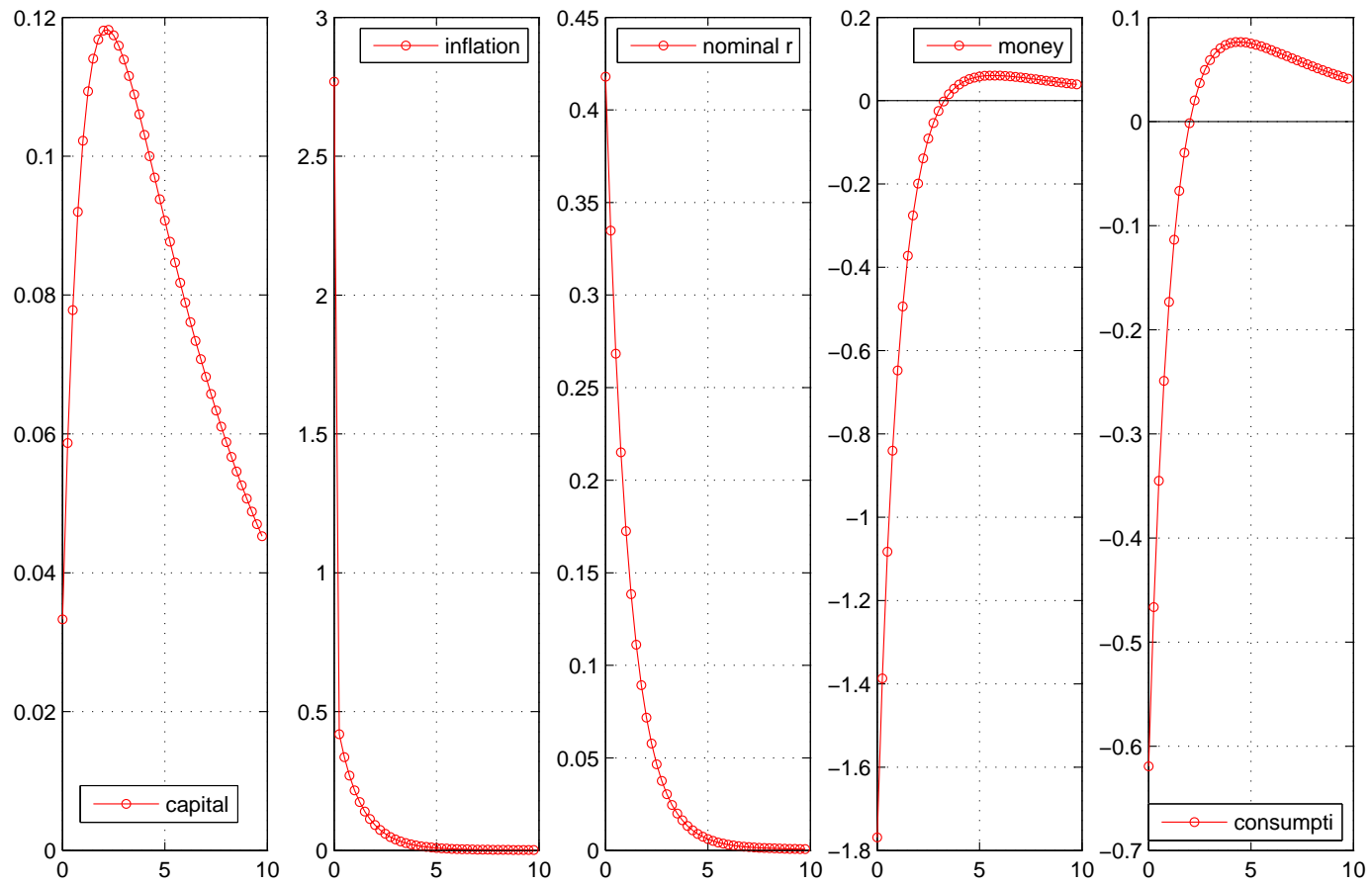
- Dynamic effects of changes in money on real activity? In general, yes, but limited. (see next slides)
- And nothing which looks like the real effects of money in the real world.
- Bottom line: Money as a medium of exchange, without nominal rigidities gives us a way of thinking about the economy, the price level, the nominal interest rate, but not much in the way of explaining fluctuations.
- Very useful however when money growth and inflation become high and variable. Turn to this.

[A simulation of the Sidrauski model]

- Utility function given by:

$$U(C_t, m_t) = \frac{1}{1 - \sigma} [(aC_t^{1-b} + (1 - a)m_t^{1-b})^{1/(1-b)}]^{1-\sigma}$$

- $1/b$ elasticity of substitution between C and m , equal to $1/16$. $\sigma = .5$, $a = .975$. ($U_{Cm} > 0$ if $b > \sigma$).
- Effects of an increase in the rate of nominal money growth of 1%, returning to normal at rate .8 per period.
- Interpretation of results. Why does consumption go down? What would happen to labor supply if it were endogenous.



3. Money growth, inflation, seignorage

Inflation and Money Growth during Seven Hyperinflations of the 1920s and 1940s

Country	Beginning	End	P_T/P_0	Av Monthly Inf rate (%)	Av Monthly M Growth (%)
Austria	Oct. 1921	Aug. 1922	70	47	31
Germany	Aug. 1922	Nov. 1923	1.0×10^{10}	322	314
Greece	Nov. 1943	Nov. 1944	4.7×10^6	365	220
Hungary 1	Mar. 1923	Feb. 1924	44	46	33
Hungary 2	Aug. 1945	Jul. 1946	3.8×10^{27}	19,800	12,200
Poland	Jan. 1923	Jan. 1924	699	82	72
Russia	Dec. 1921	Jan. 1924	1.2×10^5	57	49

P_T/P_0 : Price level in the last month of hyperinflation divided by the price level in the first month. Source: Philip Cagan, 1956.

- Was hyperinflation the result of money growth, and only money growth?
- Why was money growth so high? Did it maximize **seignorage** (revenues from money creation). And if not, then why?
- What was the role of fiscal policy?

Start with the money demand in the spirit of that we have just derived:

$$\frac{M_t}{P_t} = C_t L(r_t + \pi_t^e)$$

If money growth and inflation are high and variable, M , P and π^e will move a lot relative to C and r . So assume, for simplicity, that $C_t = C$, and $r_t = r$, so:

$$\frac{M_t}{P_t} = C L(r + \pi_t^e)$$

This gives a relation between the price level and the expected rate of inflation. The higher expected inflation, the lower real money balances, the higher the price level.

Shift to continuous time, more convenient here. Assume a particular form for the demand for money:

$$M/P = \exp(-\alpha\pi^e)$$

So, in logs:

$$m - p = -\alpha \pi^e$$

Log real money balances are a decreasing function of expected inflation. Differentiating with respect to time:

$$x - \pi = -\alpha d\pi^e/dt$$

Assume that people have adaptive expectations about expected inflation. (In an environment such as hyperinflation, this assumption makes a lot of sense. More on rational expectations below).

$$d\pi^e/dt = \beta (\pi - \pi^e)$$

Money growth and inflation

Suppose money growth is constant, at x . Will inflation converge to $\pi = x$?

Combine the two equations above to get:

$$d\pi_t^e = \frac{\beta}{1 - \alpha\beta}(x - \pi^e)$$

- If $\alpha\beta > 1$, then unstable. Why? ($x \rightarrow \pi \rightarrow d\pi^e \rightarrow \pi$, and so on)
- If $\alpha\beta < 1$, then the equilibrium is stable. Start with $x > 0$, and $\pi = 0$. Then converge to $\pi = \pi^e = x$.

Effects of an increase in x ? Jump in π above the new x , then back to new x over time. Why?

Cagan estimated α and β , found $\alpha\beta < 1$. Hyperinflation was the result of money growth, not a bubble.

Seignorage and money growth

What is the maximum revenue the government can get from money creation:

$$S \equiv \frac{dM/dt}{P} = \frac{dM/dt}{M} \frac{M}{P} = x \exp(-\alpha\pi^e)$$

So, in steady state:

$$S = x \exp(-\alpha x)$$

So $x^* = 1/\alpha$

Much lower than the growth rates of money observed during hyperinflation.

But just a steady state result. Can clearly get more in the short run, when π^e has not adjusted yet. This suggests looking at dynamics: Given seignorage, dynamics of money growth and inflation.

Start from:

$$S = x \exp(-\alpha\pi^e) \text{ or } \pi^e = \frac{1}{\alpha}(\log(x/S))$$

For a given S , draw the relation between π^e and x in π^e, x space. Concave. Can cross the 45 degree line twice, once if tangent, not at all if no way to generate the required seignorage in steady state.

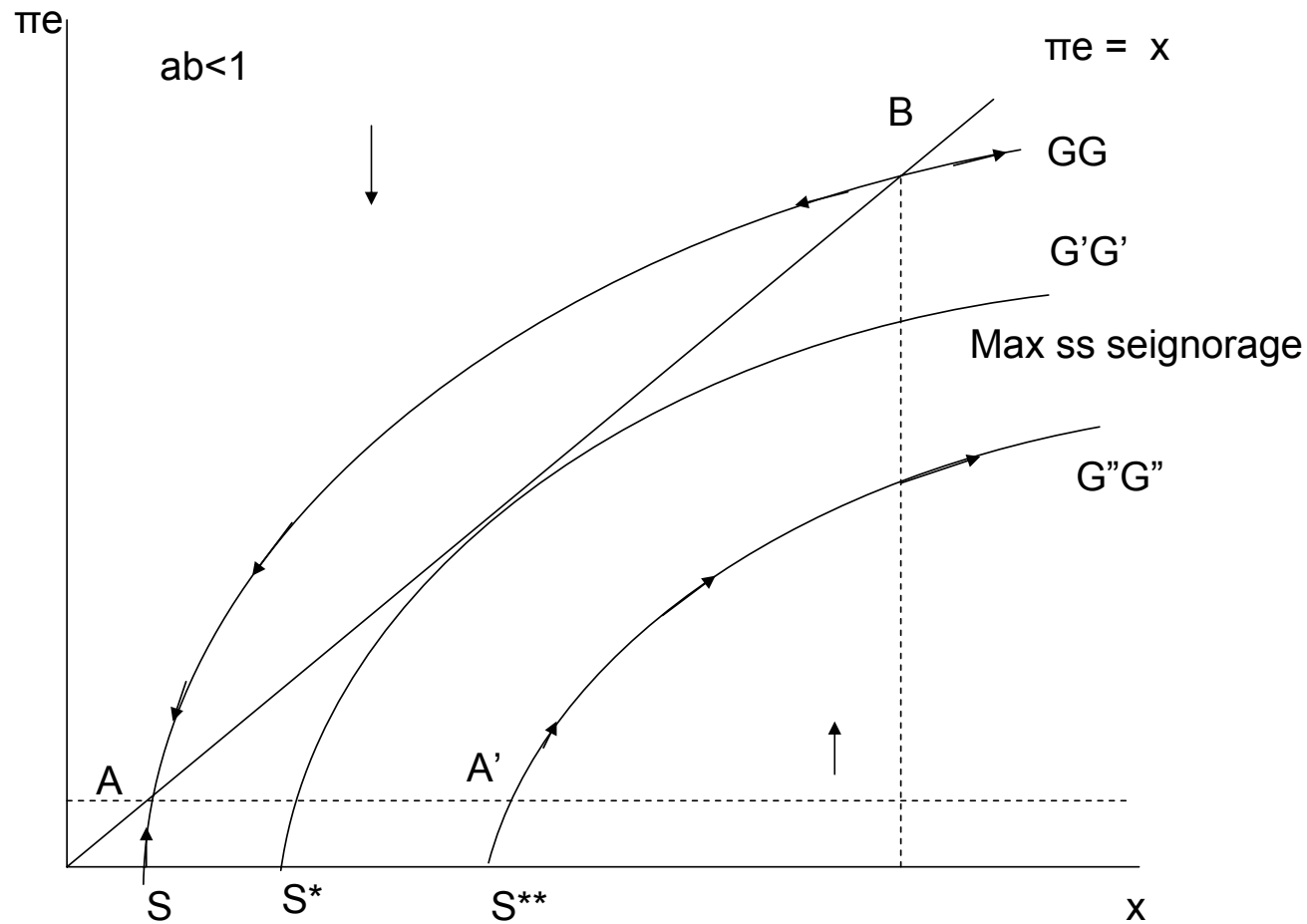
Which equilibrium is stable? Using the equation for adaptive expectations and the money demand relation in derivative form:

$$d\pi^e/dt = \beta/(1 - \alpha\beta) (x - \pi^e)$$

If $\alpha\beta < 1$, and if two equilibria, lower one is stable. Start from it (A), and suppose S increases so no equilibrium (go below $G'G'$). See Figure.

Then, money growth and inflation will keep increasing. This appears to capture what happens during hyperinflations.

Dynamics of seignorage and inflation



Nominal Money Growth and Seignorage, during hyperinflations.

	Rate of Money Growth Maximizing Seignorage (% per month)	Implied Seignorage (% of output)	Actual Rate of Money Growth (% per month)
Austria	12	13	31
Germany	20	14	314
Greece	28	11	220
Hungary 1	12	19	33
Hungary 2	32	6	12,200
Poland	54	4.6	72
Russia	39	0.5	49

Monthly rate of nominal money growth, in percent.

Some other issues

- Adaptive or rational expectations? (see BF 5-1) Recent work by Sargent. Learning.
- Fiscal policy, and the effects of inflation on the need for seignorage. (See Dornbusch et al) Inflation may itself reduce revenues.
- “Unpleasant monetarist arithmetic?” (see BF 10-2) For a given deficit, government can finance by debt or money. Which will lead to more inflation today?

Summary

- Need to introduce money for transactions. CIA constraints. but can get complicated.
- Shortcut. (Real money) in the utility function.
- Nominal interest rates affect the demand for money, real interest rates affect the slope of consumption.
- Money often superneutral. Real effects from dynamics do not resemble what is in the data.
- Money growth central to inflation, hyperinflation. The strong link between fiscal policy and monetary policy.