14.452 Economic Growth: Lecture 3, The Solow Growth Model and the Data

Daron Acemoglu

MIT

November 3, 2009.

Solow Growth Model and the Data

- Use Solow model or extensions to interpret both economic growth over time and cross-country output differences.
- Focus on *proximate causes* of economic growth.

Growth Accounting I

- Aggregate production function in its general form:

$$Y(t) = F\left[K(t), L(t), A(t)\right].$$

- Combined with competitive factor markets, gives Solow (1957) *growth accounting framework.*
- Continuous-time economy and differentiate the aggregate production function with respect to time.
- Dropping time dependence,

$$\frac{\dot{Y}}{Y} = \frac{F_A A}{Y}\frac{\dot{A}}{A} + \frac{F_K K}{Y}\frac{\dot{K}}{K} + \frac{F_L L}{Y}\frac{\dot{L}}{L}. \tag{1}$$

Growth Accounting II

- Denote growth rates of output, capital stock and labor by $g \equiv \dot{Y}/Y$, $g_K \equiv \dot{K}/K$ and $g_L \equiv \dot{L}/L$.
- Define the contribution of technology to growth as

$$x \equiv \frac{F_A A}{Y} \frac{\dot{A}}{A}$$

- Recall with competitive factor markets, $w = F_L$ and $R = F_K$.
- Define factor shares as $\alpha_K \equiv RK/Y$ and $\alpha_L \equiv wL/Y$.
- Putting all these together, (1) the *fundamental growth accounting* equation

$$x = g - \alpha_K g_K - \alpha_L g_L. \tag{2}$$

- Gives estimate of contribution of technological progress, *Total Factor Productivity* (TFP) or Multi Factor Productivity.

Growth Accounting III

- Denoting an estimate by "^":

$$\hat{x}(t) = g(t) - \alpha_K(t) g_K(t) - \alpha_L(t) g_L(t). \tag{3}$$

- All terms on right-hand side are "estimates" obtained with a range of assumptions from national accounts and other data sources.
- If interested in $\dot{A}/A$ rather than x, need further assumptions. For example, if we assume

$$Y(t) = \tilde{F}[K(t), A(t)L(t)],$$

then

$$\frac{\dot{A}}{A} = \frac{1}{\alpha_L}[g - \alpha_K g_K - \alpha_L g_L],$$

- But not particularly useful,the economically interesting object is $\hat{x}$ in (3).

Growth Accounting IV

- In continuous time, equation (3) is exact.
- With discrete time, potential problem in using (3): over the time horizon factor shares can change.
- Use beginning-of-period or end-of-period values of α_K and α_L?
  - Either might lead to seriously biased estimates.
  - Best way of avoiding such biases is to use as high-frequency data as possible.
  - Typically use factor shares calculated as the average of the beginning and end of period values.
- In discrete time, the analog of equation (3) becomes

$$\hat{x}_{t,t+1} = g_{t,t+1} - \bar{\alpha}_{K,t,t+1} g_{K,t,t+1} - \bar{\alpha}_{L,t,t+1} g_{L,t,t+1}, \tag{4}$$

- $g_{t,t+1}$ is the growth rate of output between t and $t+1$; other growth rates defined analogously.

Growth Accounting V

- Moreover,

$$\bar{\alpha}_{K,t,t+1} \equiv \frac{\alpha_K(t)+\alpha_K(t+1)}{2}$$
$$\text{and } \bar{\alpha}_{L,t,t+1} \equiv \frac{\alpha_L(t)+\alpha_L(t+1)}{2}$$

- Equation (4) would be a fairly good approximation to (3) when the difference between t and $t+1$ is small and the capital-labor ratio does not change much during this time interval.
- Solow's (1957) applied this framework to US data: a large part of the growth was due to technological progress.
- From early days, however, a number of pitfalls were recognized.
  - Moses Abramovitz (1956): dubbed the $\hat{x}$ term "the measure of our ignorance".
  - If we mismeasure g_L and g_K we will arrive at inflated estimates of $\hat{x}$.

Growth Accounting VI

- Reasons for mismeasurement:
    - what matters is not labor hours, but effective labor hours
        - important—though difficult—to make adjustments for changes in the *human capital* of workers.
    - measurement of capital inputs:
        - in the theoretical model, capital corresponds to the final good used as input to produce more goods.
        - in practice, capital is machinery, need assumptions about how relative prices of machinery change over time.
        - typical assumption was to use capital expenditures but if machines become cheaper would severely underestimate g_K

Solow Model and Regression Analyses I

- Another popular approach of taking the Solow model to data: *growth regressions*, following Barro (1991).
- Return to basic Solow model with constant population growth and labor-augmenting technological change in continuous time:

$$y(t) = A(t) f(k(t)), \tag{5}$$

and

$$\frac{\dot{k}(t)}{k(t)} = \frac{sf(k(t))}{k(t)} - \delta - g - n. \tag{6}$$

Solow Model and Regression Analyses II

- Define $y^*(t) \equiv A(t) f(k^*)$; refer to $y^*(t)$ as the "steady-state level of output per capita" even though it is not constant.
- First-order Taylor expansions of $\log y(t)$ with respect to $\log k(t)$ around $\log k^*(t)$ and manipulation of previous equations lead to (see homework):

$$\log y(t) - \log y^*(t) \simeq \varepsilon_f(k^*)(\log k(t) - \log k^*).$$

- Combining this with the previous equation, "convergence equation":

$$\frac{\dot{y}(t)}{y(t)} \simeq g - (1 - \varepsilon_f(k^*))(\delta + g + n)(\log y(t) - \log y^*(t)). \quad (7)$$

- Two sources of growth in Solow model: g, the rate of technological progress, and "convergence".

Solow Model and Regression Analyses III

- Latter source, convergence:
  - Negative impact of the gap between current level and steady-state level of output per capita on rate of capital accumulation (recall $0 < \varepsilon_f(k^*) < 1$).
  - The lower is $y(t)$ relative to $y^*(t)$, hence the lower is $k(t)$ relative to k^*, the greater is $f(k^*)/k^*$, and this leads to faster growth in the effective capital-labor ratio.
- Speed of convergence in (7), measured by the term $(1 - \varepsilon_f(k^*))(\delta + g + n)$, depends on:
  - $\delta + g + n$: determines rate at which effective capital-labor ratio needs to be replenished.
  - $\varepsilon_f(k^*)$: when $\varepsilon_f(k^*)$ is high, we are close to a linear—AK—production function, convergence should be slow.

Example: Cobb-Douglas Production Function and Converges

- Consider Cobb-Douglas production function $Y(t) = A(t) K(t)^{\alpha} L(t)^{1-\alpha}$.
- Implies that $y(t) = A(t) k(t)^{\alpha}$, $\varepsilon_f(k(t)) = \alpha$. Therefore, (7) becomes

$$\frac{\dot{y}(t)}{y(t)} \simeq g - (1-\alpha)(\delta + g + n)(\log y(t) - \log y^*(t)).$$

- Enables us to "calibrate" the speed of convergence in practice
- Focus on advanced economies
  - $g \simeq 0.02$ for approximately 2% per year output per capita growth,
  - $n \simeq 0.01$ for approximately 1% population growth and
  - $\delta \simeq 0.05$ for about 5% per year depreciation.
  - Share of capital in national income is about $1/3$, so $\alpha \simeq 1/3$.

Example (continued)

- Thus convergence coefficient would be around 0.054 ($\simeq 0.67 \times 0.08$).
- Very rapid rate of convergence:
    - gap of income between two similar countries should be halved in little more than 10 years
- At odds with the patterns we saw before.

Solow Model and Regression Analyses (continued)

- Using (7), we can obtain a growth regression similar to those estimated by Barro (1991).
- Using discrete time approximations, equation (7) yields:

$$g_{i,t,t-1} = b^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \tag{8}$$

- $\varepsilon_{i,t}$ is a stochastic term capturing all omitted influences.
- If such an equation is estimated in the sample of core OECD countries, b^1 is indeed estimated to be negative.
- But for the whole world, no evidence for a negative b^1. If anything, b^1 would be positive.
- I.e., there is no evidence of world-wide convergence,
- Barro and Sala-i-Martin refer to this as "unconditional convergence."

Solow Model and Regression Analyses (continued)

- Unconditional convergence may be too demanding:
  - requires income gap between any two countries to decline, irrespective of what types of technological opportunities, investment behavior, policies and institutions these countries have.
  - If countries do differ, Solow model would *not* predict that they should converge in income level.
- If countries differ according to their characteristics, a more appropriate regression equation may be:

$$g_{i,t,t-1} = b_i^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \tag{9}$$

- Now the constant term, b_i^0, is country specific.
- Slope term, measuring the speed of convergence, b^1, should also be country specific.
- May then model b_i^0 as a function of certain country characteristics.

Problems with Regression Analyses

- If the true equation is (9), (8) would not be a good fit to the data.
- I.e., there is no guarantee that the estimates of b^1 resulting from this equation will be negative.
- In particular, it is natural to expect that $Cov\left(b_i^0, \log y_{i,t-1}\right) < 0$:
  - economies with certain growth-reducing characteristics will have low levels of output.
  - Implies a negative bias in the estimate of b^1 in equation (8), when the more appropriate equation is (9).
- With this motivation, Barro (1991) and Barro and Sala-i-Martin (2004) favor the notion of "conditional convergence:"
  - convergence effects should lead to negative estimates of b^1 once b_i^0 is allowed to vary across countries.

Problems with Regression Analyses (continued)

- Barro (1991) and Barro and Sala-i-Martin (2004) estimate models where b_i^0 is assumed to be a function of:
  - male schooling rate, female schooling rate, fertility rate, investment rate, government-consumption ratio, inflation rate, changes in terms of trades, openness and institutional variables such as rule of law and democracy.
- In regression form,

$$g_{i,t,t-1} = \mathbf{X}'_{i,t}\boldsymbol{\beta} + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \tag{10}$$

- $\mathbf{X}_{i,t}$ is a (column) vector including the variables mentioned above (and a constant).
- Imposes that b_i^0 in equation (9) can be approximated by $\mathbf{X}'_{i,t}\boldsymbol{\beta}$.
- Conditional convergence: regressions of (10) tend to show a negative estimate of b^1.
- But the magnitude is much lower than that suggested by the computations in the Cobb-Douglas Example.

Problems with Regression Analyses (continued)

- Regressions similar to (10) have not only been used to support "conditional convergence," but also to estimate the "determinants of economic growth".
- Coefficient vector $\boldsymbol{\beta}$: information about *causal effects* of various variables on economic growth.
- Several problematic features with regressions of this form. These include:
- **Many variables in $X_{i,t}$ and $\log y_{i,t-1}$, are econometrically endogenous: jointly determined $g_{i,t,t-1}$.**
  - May argue b^1 is of interest even without "causal interpretation".
  - But if $\mathbf{X}_{i,t}$ is econometrically endogenous, estimate of b^1 will also be inconsistent (unless $\mathbf{X}_{i,t}$ is independent from $\log y_{i,t-1}$).

Problems with Regression Analyses (continued)

- **Even if $X_{i,t}$'s were econometrically exogenous, a negative b^1 could be by measurement error or other transitory shocks to $y_{i,t}$.**
- For example, suppose we only observe $\tilde{y}_{i,t} = y_{i,t} \exp\left(u_{i,t}\right)$.
  - Note

$$\log \tilde{y}_{i,t} - \log \tilde{y}_{i,t-1} = \log y_{i,t} - \log y_{i,t-1} + u_{i,t} - u_{i,t-1}.$$

  - Since measured growth is $\tilde{g}_{i,t,t-1} \approx \log \tilde{y}_{i,t} - \log \tilde{y}_{i,t-1} = \log y_{i,t} - \log y_{i,t-1} + u_{i,t} - u_{i,t-1}$, when we look at the growth regression

$$\tilde{g}_{i,t,t-1} = \mathbf{X}'_{i,t}\boldsymbol{\beta} + b^1 \log \tilde{y}_{i,t-1} + \varepsilon_{i,t},$$

  - measurement error $u_{i,t-1}$ will be part of both $\varepsilon_{i,t}$ and $\log \tilde{y}_{i,t-1} = \log y_{i,t-1} + u_{i,t-1}$: negative bias in the estimation of b^1.
  - Thus can end up negative estimate of b^1, *even when* there is no conditional convergence.

Problems with Regression Analyses (continued)

- **Interpretation of regression equations like (10) is not always straightforward**
  - Investment rate in $\mathbf{X}_{i,t}$: in Solow model, differences in investment rates are *the* channel for convergence.
  - Thus conditional on investment rate, there should be no further effect of gap between current and steady-state level of output.
  - Same concern for variables in $\mathbf{X}_{i,t}$ that would affect primarily by affecting investment or schooling rate.
- **Equation for (7) is derived for closed Solow economy.**

The Solow Model with Human Capital I

- Labor hours supplied by different individuals do not contain the same efficiency units.
- Focus on the continuous time economy and suppose:

$$Y = F(K, H, AL), \tag{11}$$

  where H denotes "human capital".
- Assume throughout that $A > 0$.
- Assume $F : \mathbb{R}^3_+ \to \mathbb{R}_+$ in (11) is twice continuously differentiable in K, H and L, and satisfies the equivalent of the neoclassical assumptions.
- Households save a fraction s_k of their income to invest in physical capital and a fraction s_h to invest in human capital.
- Human capital also depreciates in the same way as physical capital, denote depreciation rates by δ_k and δ_h.

The Solow Model with Human Capital III

- Assume constant population growth and a constant rate of labor-augmenting technological progress, i.e.,

$$\frac{\dot{L}(t)}{L(t)} = n \text{ and } \frac{\dot{A}(t)}{A(t)} = g.$$

- Defining effective human and physical capital ratios as

$$k(t) \equiv \frac{K(t)}{A(t)L(t)} \text{ and } h(t) \equiv \frac{H(t)}{A(t)L(t)},$$

- Using the constant returns to scale, output per effective unit of labor can be written as

$$\begin{aligned} \hat{y}(t) &\equiv \frac{Y(t)}{A(t)L(t)} \\ &= F\left(\frac{K(t)}{A(t)L(t)}, \frac{H(t)}{A(t)L(t)}, 1\right) \\ &\equiv f(k(t), h(t)). \end{aligned}$$

The Solow Model with Human Capital IV

- Law of motion of $k(t)$ and $h(t)$ can then be obtained as:

$$\begin{aligned} \dot{k}(t) &= s_k f(k(t), h(t)) - (\delta_k + g + n) k(t), \\ \dot{h}(t) &= s_h f(k(t), h(t)) - (\delta_h + g + n) h(t). \end{aligned}$$

- Steady-state equilibrium: effective human and physical capital ratios, (k^*, h^*), which satisfiy:

$$s_k f(k^*, h^*) - (\delta_k + g + n) k^* = 0, \tag{12}$$

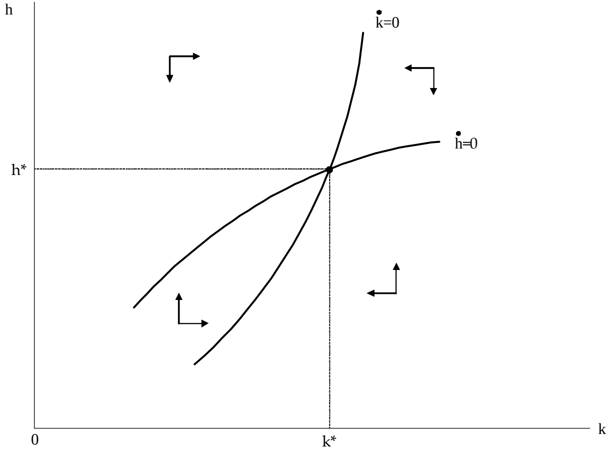
and

$$s_h f(k^*, h^*) - (\delta_h + g + n) h^* = 0. \tag{13}$$

The Solow Model with Human Capital V

- Focus on steady-state equilibria with $k^* > 0$ and $h^* > 0$ (if $f(0,0) = 0$, then there exists a trivial steady state with $k = h = 0$, which we ignore it).
- Can first prove that steady-state equilibrium is unique. To see this heuristically, consider the Figure in the (k, h) space.
- Both lines are upward sloping, but proof of next proposition shows (13) is always shallower in the (k, h) space, so the two curves can only intersect once.

Proposition In the augmented Solow model with human capital, there exists a unique, globally stable steady-state equilibrium (k^*, h^*).



Courtesy of Princeton University Press. Used with permission.
Figure 3.1 in Acemoglu, Daron. *Introduction to Modern Economic Growth*. Princeton, NJ: Princeton University Press, 2009.
ISBN: 9780691132921.

Figure: Dynamics of physical capital-labor and human capital-labor ratios in the Solow model with human capital.

Example: Cobb-Douglas Production Function

- Aggregate production function is

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t) L(t))^{1-\alpha-\beta}, \tag{14}$$

where $0 < \alpha < 1$, $0 < \beta < 1$ and $\alpha + \beta < 1$.

- Output per effective unit of labor can then be written as

$$\hat{y}(t) = k^{\alpha}(t) h^{\beta}(t),$$

with the same definition of $\hat{y}(t)$, $k(t)$ and $h(t)$ as above.

Example (continued)

- Using this functional form, (12) and (13) give the unique steady-state equilibrium:

$$
\begin{aligned}
k^* &= \left(\left(\frac{s_k}{n+g+\delta_k} \right)^{1-\beta} \left(\frac{s_h}{n+g+\delta_h} \right)^{\beta} \right)^{\frac{1}{1-\alpha-\beta}} \qquad (15) \\
h^* &= \left(\left(\frac{s_k}{n+g+\delta_k} \right)^{\alpha} \left(\frac{s_h}{n+g+\delta_h} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\beta}},
\end{aligned}
$$

- Higher saving rate in physical capital not only increases k^*, but also h^*.
- Same applies for a higher saving rate in human capital.
- Reflects that higher k^* raises overall output and thus the amount invested in schooling (since s_h is constant).

Example (continued)

- Given (15), output per effective unit of labor in steady state is obtained as

$$\hat{y}^* = \left(\frac{s_k}{n+g+\delta_k}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{s_h}{n+g+\delta_h}\right)^{\frac{\alpha}{1-\alpha-\beta}}. \tag{16}$$

- Relative contributions of the saving rates depends on the shares of physical and human capital:
  - the larger is β, the more important is s_k and the larger is α, the more important is s_h.

A World of Augmented Solow Economies I

- Mankiw, Romer and Weil (1992) used regression analysis to take the augmented Solow model, with human capital, to data.
- Use the Cobb-Douglas model and envisage a world consisting of $j = 1, ..., N$ countries.
- "Each country is an island": countries do not interact (perhaps except for sharing some common technology growth).
- Country $j = 1, ..., N$ has the aggregate production function:

$$Y_j(t) = K_j(t)^{\alpha} H_j(t)^{\beta} (A_j(t) L_j(t))^{1-\alpha-\beta}.$$

- Nests the basic Solow model without human capital when $\alpha = 0$.
- Countries differ in terms of their saving rates, $s_{k,j}$ and $s_{h,j}$, population growth rates, n_j, and technology growth rates $\dot{A}_j(t)/A_j(t) = g_j$.
- Define $k_j \equiv K_j/A_jL_j$ and $h_j \equiv H_j/A_jL_j$.

A World of Augmented Solow Economies II

- Focus on a world in which each country is in their steady state
- Equivalents of equations (15) apply here and imply:

$$k_j^* = \left(\left(\frac{s_{k,j}}{n_j + g_j + \delta_k} \right)^{1-\beta} \left(\frac{s_{h,j}}{n_j + g_j + \delta_h} \right)^{\beta} \right)^{\frac{1}{1-\alpha-\beta}}$$

$$h_j^* = \left(\left(\frac{s_{k,j}}{n_j + g_j + \delta_k} \right)^{\alpha} \left(\frac{s_{h,j}}{n_j + g_j + \delta_h} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\beta}} .$$

- Consequently, using (16), the "steady-state"/balanced growth path income per capita of country j can be written as

$$y_j^*(t) \equiv \frac{Y(t)}{L(t)} = A_j(t) \left(\frac{s_{k,j}}{n_j + g_j + \delta_k} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{s_{h,j}}{n_j + g_j + \delta_h} \right)^{\frac{\beta}{1-\alpha-\beta}} . \quad (17)$$

A World of Augmented Solow Economies II

- Here $y_j^*(t)$ stands for output per capita of country j along the balanced growth path.
- Note if g_j's are not equal across countries, income per capita will diverge.
- Mankiw, Romer and Weil (1992) make the following assumption:

$$A_j(t) = \bar{A}_j \exp(gt).$$

- Countries differ according to technology *level*, (initial level $\bar{A}_j$) but they share the same common technology growth rate, g.

A World of Augmented Solow Economies III

- Using this together with (17) and taking logs, equation for the balanced growth path of income for country $j = 1, ..., N$:

$$\ln y_j^*(t) = \ln \bar{A}_j + gt + \frac{\alpha}{1-\alpha-\beta} \ln\left(\frac{s_{k,j}}{n_j + g + \delta_k}\right) + \frac{\beta}{1-\alpha-\beta} \ln\left(\frac{s_{h,j}}{n_j + g + \delta_h}\right). \tag{18}$$

- Mankiw, Romer and Weil (1992) take:
  - $\delta_k = \delta_h = \delta$ and $\delta + g = 0.05$.
  - $s_{k,j}$=average investment rates (investments/GDP).
  - $s_{h,j}$=fraction of the school-age population that is enrolled in secondary school.

A World of Augmented Solow Economies IV

- Even with all of these assumptions, (18) can still not be estimated consistently.
- $\ln \bar{A}_j$ is unobserved (at least to the econometrician) and thus will be captured by the error term.
- Most reasonable models would suggest $\ln \bar{A}_j$'s should be correlated with investment rates.
- Thus an estimation of (18) would lead to omitted variable bias and inconsistent estimates.
- Implicitly, MRW make another *crucial* assumption, the **orthogonal technology assumption**:

$$\bar{A}_j = \varepsilon_j A, \text{ with } \varepsilon_j \text{ orthogonal to all other variables.}$$

Cross-Country Income Differences: Regressions I

- MRW first estimate equation (18) without the human capital term for the cross-sectional sample of non-oil producing countries

$$\ln y_j^* = \text{constant} + \frac{\alpha}{1-\alpha}\ln\left(s_{k,j}\right) - \frac{\alpha}{1-\alpha}\ln\left(n_j + g + \delta_k\right) + \varepsilon_j.$$

Cross-Country Income Differences: Regressions II

Estimates of the Basic Solow Model

	MRW	Updated data	
	1985	1985	2000
$\ln(s_k)$	1.42	1.01	1.22
	(.14)	(.11)	(.13)
$\ln(n+g+\delta)$	-1.97	-1.12	-1.31
	(.56)	(.55)	(.36)
Adj R^2	.59	.49	.49
Implied α	.59	.50	.55
No. of observations	98	98	107

Table 3.1 in Acemoglu, Daron. *Introduction to Modern Economic Growth* Princeton, NJ: Princeton University Press, 2009. ISBN: 9780691132921.

Cross-Country Income Differences: Regressions III

- Their estimates for $\alpha / (1-\alpha)$, implies that α must be around $2/3$, but should be around $1/3$.
- The most natural reason for the high implied values of α is that ε_j is correlated with $\ln(s_{k,j})$, either because:
  - the orthogonal technology assumption is not a good approximation to reality or
  - there are also human capital differences correlated with $\ln(s_{k,j})$.
- Mankiw, Romer and Weil favor the second interpretation and estimate the augmented model,

$$\begin{aligned} \ln y_j^* &= \text{cst} + \frac{\alpha}{1-\alpha-\beta} \ln(s_{k,j}) - \frac{\alpha}{1-\alpha-\beta} \ln(n_j + g + \delta_k) \\ &+ \frac{\beta}{1-\alpha-\beta} \ln(s_{h,j}) - \frac{\beta}{1-\alpha-\beta} \ln(n_j + g + \delta_h) + \varepsilon_j . \end{aligned} \quad (19)$$

Estimates of the Augmented Solow Model

	MRW	Updated data	
	1985	1985	2000
$\ln(s_k)$	.69	.65	.96
	(.13)	(.11)	(.13)
$\ln(n+g+\delta)$	-1.73	-1.02	-1.06
	(.41)	(.45)	(.33)
$\ln(s_h)$	.66	.47	.70
	(.07)	(.07)	(.13)
Adj R^2	.78	.65	.60
Implied α	.30	.31	.36
Implied β	.28	.22	.26
No. of observations	98	98	107

Table 3.2 in Acemoglu, Daron. *Introduction to Modern Economic Growth*. Princeton, NJ: Princeton University Press, 2009. ISBN: 9780691132921.

Cross-Country Income Differences: Regressions IV

- If these regression results are reliable, they give a big boost to the augmented Solow model.
    - Adjusted R^2 suggests that three quarters of income per capita differences across countries can be explained by differences in their physical and human capital investment.
- Immediate implication is technology (TFP) differences have a somewhat limited role.
- But this conclusion should not be accepted without further investigation.

Challenges to Regression Analyses I

- **Technology differences across countries are not orthogonal to all other variables.**
- $\bar{A}_j$ is correlated with measures of s_j^h and s_j^k for two reasons.
  - *omitted variable bias*: societies with high $\bar{A}_j$ will be those that have invested more in technology for various reasons; same reasons likely to induce greater investment in physical and human capital as well.
  - *reverse causality*: complementarity between technology and physical or human capital imply that countries with high $\bar{A}_j$ will find it more beneficial to increase their stock of human and physical capital.
- In terms of (19), implies that key right-hand side variables are correlated with the error term, ε_j.
- OLS estimates of α and β and R^2 are biased upwards.

Challenges to Regression Analyses II

- **α is too large relative to what we should expect on the basis of microeconometric evidence.**
- The working age population enrolled in school ranges from 0.4% to over 12% in the sample of countries.
- Predicted log difference in incomes between these two countries is

$$\frac{\beta}{1-\alpha-\beta}\left(\ln 12-\ln\left(0.4\right)\right)=0.66\times\left(\ln 12-\ln\left(0.4\right)\right)\approx 2.24.$$

- Thus a country with schooling investment of over 12 should be about $\exp\left(2.24\right)-1\approx 8.5$ times richer than one with investment of around 0.4.

Challenges to Regression Analyses III

- Take Mincer regressions of the form:

$$\ln w_i = \mathbf{X}_i'\gamma + \phi S_i, \tag{20}$$

- Microeconometrics literature suggests that ϕ is between 0.06 and 0.10.
- Can deduce how much richer a country with 12 if we assume:
  - That the micro-level relationship as captured by (20) applies identically to all countries.
  - That there are no *human capital externalities*.

Challenges to Regression Analyses IV

- Suppose that each firm f in country j has access to the production function
$$y_{fj} = K_f^{\alpha} \left(A_j H_f\right)^{1-\alpha},$$
- Suppose also that firms in this country face a cost of capital equal to R_j. With perfectly competitive factor markets,
$$R_j = \alpha \left(\frac{K_f}{A_j H_f}\right)^{-(1-\alpha)}. \tag{21}$$
- Implies all firms ought to function at the same physical to human capital ratio.
- Thus all workers, irrespective of level of schooling, ought to work at the same physical to human capital ratio.

Challenges to Regression Analyses V

- Another direct implication of competitive labor markets is that in country j,
$$w_j = (1-\alpha)\,\alpha^{\alpha/(1-\alpha)} A_j R_j^{-\alpha/(1-\alpha)}.$$
- Consequently, a worker with human capital h_i will receive a wage income of $w_j h_i$.
- Next, substituting for capital from (21), we have total income in country j as
$$Y_j = \alpha^{\alpha/(1-\alpha)} A_j R_j^{-\alpha/(1-\alpha)} H_j,$$
where H_j is the total efficiency units of labor in country j.

Challenges to Regression Analyses V

- Implies that ceteris paribus (in particular, holding constant capital intensity corresponding to R_j and technology, A_j), a doubling of human capital will translate into a doubling of total income.
- It may be reasonable to keep technology, A_j, constant, but R_j may change in response to a change in H_j.
  - Maybe, but second-order:
    - International capital flows may work towards equalizing the rates of returns across countries.
    - When capital-output ratio is constant, which Uzawa Theorem established as a requirement for a balanced growth path, then R_j will indeed be constant
- So in the absence of *human capital externalities*: a country with 12 more years of average schooling should have between $\exp(0.10 \times 12) \simeq 3.3$ and $\exp(0.06 \times 12) \simeq 2.05$ times the stock of human capital of a county with fewer years of schooling.

Challenges to Regression Analyses VI

- Thus holding other factors constant, this country should be about 2-3 times as rich as the country with zero years of average schooling.
- Much less than the 8.5 fold difference implied by the Mankiw-Romer-Weil analysis.
- Thus β in MRW is too high relative to the estimates implied by the microeconometric evidence and thus likely upwardly biased.
- Overestimation of α is, in turn, most likely related to correlation between the error term ε_j and the key right-hand side regressors in (19).

Calibrating Productivity Differences I

- Suppose each country has access to the Cobb-Douglas aggregate production function:

$$Y_j = K_j^{\alpha} \left(A_j H_j\right)^{1-\alpha}, \tag{22}$$

- Each worker in country j has S_j years of schooling.
- Then using the Mincer equation (20) ignoring the other covariates and taking exponents, H_j can be estimated as

$$H_j = \exp\left(\phi S_j\right) L_j,$$

- Does not take into account differences in other "human capital" factors, such as experience.

Calibrating Productivity Differences II

- Let the rate of return to acquiring the Sth year of schooling be $\phi(S)$.
- A better estimate of the stock of human capital can be constructed as

$$H_j = \sum_S \exp\{\phi(S)\, S\}\, L_j(S)$$

- $L_j(S)$ now refers to the total employment of workers with S years of schooling in country j.
- Series for K_j can be constructed from Summers-Heston dataset using investment data and the perpetual inventory method.

$$K_j(t+1) = (1-\delta)\, K_j(t) + I_j(t),$$

- Assume, following Hall and Jones that $\delta = 0.06$.
- With same arguments as before, choose a value of $1/3$ for α.

Calibrating Productivity Differences III

- Given series for H_j and K_j and a value for α, construct "predicted" incomes at a point in time using

$$\hat{Y}_j = K_j^{1/3} \left(A_{US} H_j\right)^{2/3}$$

- A_{US} is computed so that $Y_{US} = K_{US}^{1/3} \left(A_{US} H_{US}\right)^{2/3}$.
- Once a series for $\hat{Y}_j$ has been constructed, it can be compared to the actual output series.
- Gap between the two series represents the contribution of technology.
- Alternatively, could back out country-specific technology terms (relative to the United States) as

$$\frac{A_j}{A_{US}} = \left(\frac{Y_j}{Y_{US}}\right)^{3/2} \left(\frac{K_{US}}{K_j}\right)^{1/2} \left(\frac{H_{US}}{H_j}\right).$$

Calibrating Productivity Differences IV

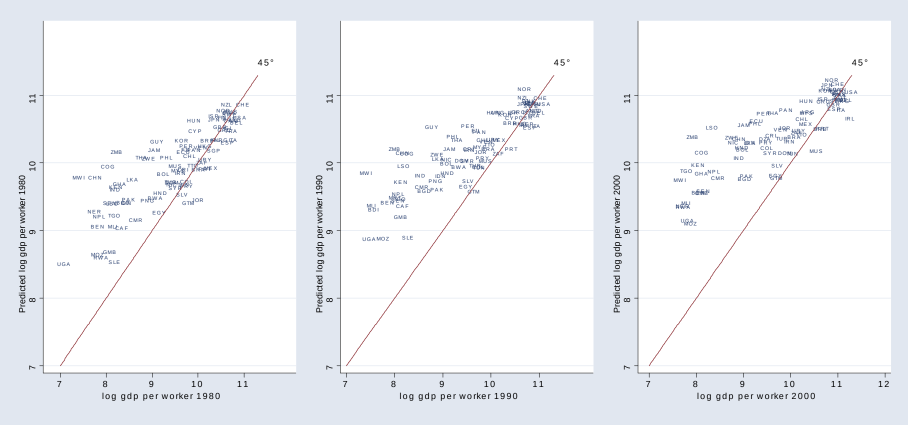


Figure 3.2 in Acemoglu, Daron. *Introduction to Modern Economic Growth*. Princeton, NJ: Princeton University Press, 2009. ISBN: 9780691132921.

Figure: Calibrated technology levels relative to the US technology (from the Solow growth model with human capital) versus log GDP per worker, 1980, 1990 and 2000.

Calibrating Productivity Differences V

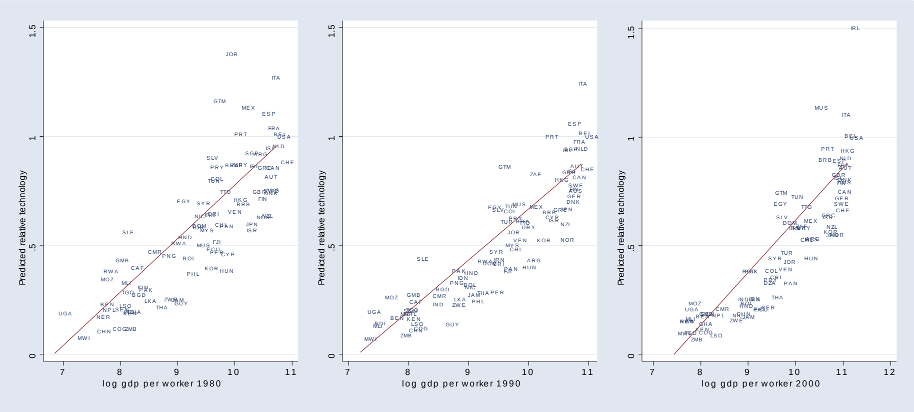


Figure 3.3 in Acemoglu, Daron. *Introduction to Modern Economic Growth*. Princeton, NJ: Princeton University Press, 2009. ISBN: 9780691132921.

Figure: Calibrated technology levels relative to the US technology (from the Solow growth model with human capital) versus log GDP per worker, 1980, 1990 and 2000.

Calibrating Productivity Differences VI

The following features are noteworthy:

- Differences in physical and human capital still matter a lot.
- However, differently from the regression analysis, this exercise also shows significant *technology (productivity) differences*.
- Same pattern visible in the next three figures for the estimates of the technology differences, A_j / A_{US}, against log GDP per capita in the corresponding year.
- Also interesting is the pattern that the empirical fit of the neoclassical growth model seems to deteriorate over time.

Challenges to Callibration I

- In addition to the standard assumptions of competitive factor markets, we had to assume :
  - no human capital externalities, a Cobb-Douglas production function, and a range of approximations to measure cross-country differences in the stocks of physical and human capital.
- The calibration approach is in fact a close cousin of the growth-accounting exercise (sometimes referred to as "levels accounting").
- Imagine that the production function that applies to all countries in the world is

$$F(K_j, H_j, A_j),$$

- Assume countries differ according to their physical and human capital as well as technology—but not according to F.

Challenges to Callibration II

- Rank countries in descending order according to their physical capital to human capital ratios, K_j / H_j Then

$$\hat{x}_{j,j+1} = g_{j,j+1} - \bar{\alpha}_{K,j,j+1} g_{K,j,j+1} - \bar{\alpha}_{Lj,j+1} g_{H,j,j+1}, \tag{23}$$

- where:
  - $g_{j,j+1}$: proportional difference in output between countries j and $j+1$,
  - $g_{K,j,j+1}$: proportional difference in capital stock between these countries and
  - $g_{H,j,j+1}$: proportional difference in human capital stocks.
  - $\bar{\alpha}_{K,j,j+1}$ and $\bar{\alpha}_{Lj,j+1}$: average capital and labor shares between the two countries.
- The estimate $\hat{x}_{j,j+1}$ is then the proportional TFP difference between the two countries.

Challenges to Callibration III

- Levels-accounting faces two challenges.
  1. Data on capital and labor shares across countries are not widely available. Almost all exercises use the Cobb-Douglas approach (i.e., a constant value of α_K equal to $1/3$).
  2. The differences in factor proportions, e.g., differences in K_j / H_j, across countries are large. An equation like (23) is a good approximation when we consider small (infinitesimal) changes.

Conclusions

- Message is somewhat mixed.
  - On the positive side, despite its simplicity, the Solow model has enough substance that we can take it to data in various different forms, including TFP accounting, regression analysis and calibration.
  - On the negative side, however, no single approach is entirely convincing.
- Complete agreement is not possible, but safe to say that consensus favors the interpretation that cross-country differences in income per capita cannot be understood solely on the basis of differences in physical and human capital
- Differences in TFP are not necessarily due to technology in the narrow sense.
- Have not examined *fundamental causes* of differences in prosperity: why some societies make choices that lead them to low physical capital, low human capital and inefficient technology and thus to relative poverty.

14.452 Economic Growth
Fall 2009