14.451: Introduction to Economic Growth
Problem Set 1

Due date: February 21, 2007.

Question 1: Consider the discrete-time Solow growth model with no population growth, no technological change, constant depreciation rate of δ and a constant savings rate s. Assume that the per capita production function is given by the following non-neoclassical function:

$$f(k) = \begin{cases} A_1 k & \text{if } k \leq \bar{k} \\ A_2 k & \text{if } k > \bar{k} \end{cases}$$

where $A_2 < A_1$.

1. Explain why this production function is non-neoclassical.
2. Show that if there exist (k_1, k_2) such that $k_1 < \bar{k} < k_2$ and
$$k_2 = sA_1k_1 + (1-\delta)k_1$$
and
$$k_1 = sA_2k_2 + (1-\delta)k_2,$$
then there exists an asymptotic cycle, in which the economy never settles into a steady state and fluctuates between k_1 and k_2.
3. Can there also exist a stable steady state when such a cycle exists?
4. Show that such a cycle is not possible in the continuous-time Solow growth model for any (possibly non-neoclassical) production function $f(k)$.
5. What does the result in part 4 imply for the approximations of discrete time by continuous time as in the lecture notes?
6. In light of your answer to part 4, what do you think of the cycles in part 2?

Question 2: Read the paper "A Contribution to the Empirics of Economic Growth" by Mankiw, Romer and Weil (in the Quarterly Journal of Economics, May 1992). The dataset for the paper is given at the site below. You can copy and paste it into Excel.

http://www.economics.harvard.edu/faculty/mankiw/data.html

1. Replicate the estimation of the augmented Solow model for the non-oil sample (first column of Table II, page 420). You may use more recently updated data if you wish. State the identification assumption(s).

2. State the highest and lowest enrollment rates in the sample. According to your regression estimates, how many times richer is the country with the highest enrollment rate relative to the country with the lowest rate? Hold other variables constant.

Now consider a calibration approach to growth accounting. We use the production function:

$$Y_i = A_i K_i^{\alpha} H_i^{1-\alpha}.$$

where A_i does not grow, and human capital follows the accumulation equation as in Mankiw-Romer-Weil:

$$\dot{H}_i = s_{h,i} Y_i - \delta H_i.$$

We use a specification of human capital similar to Hall and Jones (1999):

$$H_i = e^{\phi S_i} L_i \Rightarrow \frac{H_i}{L_i} = e^{\phi S_i},$$

where S_i is average years of total schooling.

3. From the production function, what is the marginal product of human capital? What is the payment to a worker with human capital level h_i?

4. Write output per worker, $y_i \equiv \frac{Y_i}{L_i}$, in terms of $k_i \equiv \frac{K_i}{L_i}$ and $h_i \equiv \frac{H_i}{L_i}$. Derive the equation for this model corresponding to equation (12) in the Mankiw-Romer-Weil paper.

5. Now consider the specification for human capital. Use the Barro-Lee dataset (**http://www.nber.org/data/**) to construct values of $\frac{H_i}{L_i}$ for the countries in the Mankiw-Romer-Weil sample. Let $\phi = 0.08$ (in the middle of the range of empirical estimates). Use the new human capital series, and the old series for investment and population growth, to calculate predicted values of log output per worker for each country (use the last equation derived in part 4 and ignore the term in $\ln A_i$). Set the capital share $\alpha = 0.35$ and $\delta = 0.03$.

   Note: You will have to use average years of schooling for the population rather than just for workers.

6. Regress the observed log output per worker on the predicted values. Report your adjusted-R^2. Comment. Compare to the Mankiw-Romer-Weil result.

7. Finally, estimate the last equation derived in part 4 using the new human capital series. Test whether the coefficient on $\ln h_i$ is equal to the value

predicted in part 4. Report the implied value of α, and the new adjusted-R^2.

Bonus: Run a regression imposing model predictions for coefficients (except α) and any relationships between the coefficients. Report the implied α.