

VALUATION USING HOUSEHOLD PRODUCTION

14.42 LECTURE PLAN 15: APRIL 14, 2011

Hunt Allcott

PASTURE 1: ONE SITE

- Introduce intuition via PowerPoint slides
- Draw demand curve with “nearby” and “far away” towns.

Question: Why do I say “Demand Curve” in quotes?

Answer: This gives the intuitive identification, but it does not tell us about welfare.

Left Board 1: Model:

This model straight from Kolstad Chapter 9:

Maximize utility subject to time and budget constraints.

$\text{Max}_{x,v} U(x,v)$

v =visits to park

x =numeraire good.

Price of $x = 1$

Price of visiting the park = f

Budget Constraint

s.t. $wL = x + fv$

w =wage rate

L =Labor hours

Time Constraint:

$T = L + t_t v$

t_t =Travel time to the park

T = total time endowment

- What’s weird about this? The consumer is either working or traveling to parks at all times.
- How to deal with it? Add another variable for other use of leisure time. Make it exogenous: the key is that park time trades off with work time, so it’s costly.

Substitute time constraint into budget constraint:

$wT = x + [wt_t + f]v$

$= x + [p_t + f]v$

p_t = Time Cost per visit = wt_t

$= x + p_v v$

p_v =Total cost per visit= $p_t + f$

p_v is the per-visit cost. That's all that matters to determining demand. Notice that what's nice is that a dollar of entry fee is equivalent to a dollar in travel cost.

The maximization gives a demand function for park visits that depends on the total price:

$$v = v(p_v, y) = v(p_t + f, y)$$

$y = \text{Income}$

Question: Now, how do I go from distance to prices?

Answer: I need w : the opportunity cost of travel time.

Right Board 1:

Draw a graph of (Distance=Price) on the y-axis and Visitation/(Person*Year) on the x-axis.

- PowerPoint slides

Question: Is the wage rate an appropriate way of translating distance to travel cost?

- Hours may be fixed
 - So the tradeoff is with leisure time. Need to know the shadow price of leisure time.
- People may have different utility or disutility from traveling vs. working

Question: Is this consumer surplus from the park?

No: We now assume that this same demand curve applies for all cities, and aggregate over the population in each city.

Question: Is it reasonable to assume that all cities have the same demand curve?

- Sorting: people live near outdoors if they like to be outdoors
- Bigger cities further away but have higher incomes

Question: If cities have different demand curves, does that affect the quality of our demand curve estimates?

Answer: the fact that the curve doesn't fit perfectly is a natural result of the cities having different demand curves. We're only biased if the variation in demand is correlated with distance to the park.

Push Question 1: Could the variation in demand curves be correlated with distance to the park?

Answer: Yes

Push Question 2: What does this mean for the validity of our estimator?

Answer: Omitted variables bias

PASTURE 2: TWO SITES

PowerPoint: Add the Presidentials

Question: Why is demand higher for the Presidentials?

- Higher mountains
- More open space
- Skiing is possible

Question: Do we know which factor explains this?

Answer: No. But we might want to know.

PASTURE 3: LOGIT MODEL

Left Board 2: Logit Model

Question: Let's say we wanted to estimate these demand functions in order to carry out welfare analysis. What's the problem?

Push question: What's the dimensionality of this problem?

Answer: Number of parameters grows exponentially with the number of sites!

Setup:

Individuals indexed by i , Sites indexed by j .

Assume all consumers homogeneous up to a constant

Divide utility into two parts: homogeneous utility and individual-specific error

$$u_{ij} = \delta_j + \varepsilon_{ij}$$

δ_j = Homogeneous average utility

ε_{ij} = Unobserved random error

$$1[\text{Consumer } i \text{ Chooses site } j] = u_j > u_k, \text{ all } k$$

Logistic

Question: Say there is no variation in ε across consumers. What happens?

Answer: Everybody chooses the same product. So we need the error.

We transform this into a demand function by assuming a convenient distribution of ε : Logistic.

- PowerPoint slide: logistic distribution.

Consider the choice of one site vs. an "outside option"

Normalize utility of outside option to 0.

$$\Pr(\text{Monadnock}) = \Pr(u_M > 0)$$

- Comment: this is a binary choice model – like whether or not I go to college, or whether I work in fishing sector vs. agriculture sector, etc.

$$\begin{aligned}
 &= \int 1[\delta_M + \varepsilon > 0] f(\varepsilon) d\varepsilon \\
 &= \int 1[\varepsilon > -\delta_M] f(\varepsilon) d\varepsilon \\
 &= \int_{-\delta}^{\infty} f(\varepsilon) d\varepsilon \\
 &= 1 - F(-\delta)
 \end{aligned}$$

Assume F is the logistic CDF. This gives a convenient functional form!

$$\begin{aligned}
 &= 1 - 1/(1+e^\delta) \\
 &= e^\delta/(1+e^\delta)
 \end{aligned}$$

- Intuition here: Higher δ means higher probability of Monadnock.

$$\begin{aligned}
 \text{Pr(Outside Option)} &= 1 - \text{Pr(Monadnock)} = 1 - e^\delta/(1+e^\delta) \\
 &= 1/(1+e^\delta)
 \end{aligned}$$

If all consumers homogeneous, then these probabilities can be turned into market shares:

$$s_M = \text{Pr(Monadnock)} = e^\delta/(1+e^\delta) = e^\delta s_0$$

s_0 = Outside option share

$$s_M/s_0 = e^{\delta_M}$$

$$\log s_M - \log s_0 = \delta_M$$

- Point to the logistic distribution on the PowerPoint slide

Multiple Sites:

The whole point of this was to say something about multiple sites. Skip math, but under appropriate distributional assumptions for ε_{ij} , we have:

$$\text{Pr}_j = e^{\delta_j} / (1 + \sum_{k=1}^K e^{\delta_k})$$

where k indexes choices and K is the number of choices.

The set of k is called the choice set. It is mutually exclusive and exhaustive.

Outside option is defined k=0

A couple lines of algebra give an equation for the market share for all choices as a function of δ :

$$\log s_j - \log s_0 = \delta_j$$

Pause here. This is a key intermediate result. But:

- Delta isn't really of much interest. We can't do anything with it per se. We really want price elasticity and utility from different amenities.
- Plus although the dimensionality of the problem is now linear, we can reduce it further.

Characteristic Space

- PowerPoint slide

Model δ as a function of characteristics

What characteristics?

- Height
- Length of hiking trails
- Total open space

$$u_{ij} = \delta_j + \varepsilon_{ij} = \beta X_j - \eta p_{vj} + \xi_j + \varepsilon_{ij}$$

X_j = Attributes of the site j

PASTURE 4: EMPIRICAL EXAMPLE OF DISCRETE CHOICE ESTIMATION

Linear regression example.

Say that this only works under a technical assumption: that the variance of the error terms is the same.

(Otherwise have to do something more complicated, the GMM or ML model)

$$\log s_j - \log s_0 = \beta X_j - \eta p_{vj} + \xi_j$$

Use this to estimate β and η .

- PowerPoint slides on data and estimation

What concerns?

- True functional form not linear!
- ξ correlated with characteristics!
- ξ correlated with price!

What else is this useful for? Modeling demand for many other types of discrete choices:

- Yogurt. What attributes?
 - Amount of sugar
 - Fruit/plain
 - Amount of fat
 - Domestic/Imported
 - Price
- Cars. What attributes?
 - Horsepower
 - Weight
 - MPG
 - Price

Welfare

Consumer surplus for one consumer:

$$CS_i = (1/\eta) E[\max_j (\delta_j + \varepsilon_{ij})]$$

Consumer surplus over distribution of ε :

$$E(\text{CS}) = (1/\eta) \ln (\sum_j e^{\delta_j}) + \text{Constant}$$

Counterfactuals:

Notice that δ is really a function of observable attributes

We can replace the log-sum term with a different choice set and determine consumer surplus as the difference in the two scenarios!

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14.42 / 14.420 Environmental Policy and Economics
Spring 2011

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