Problem Set #1
14.41 Public Economics

DUE: September 24, 2010

1 Question One

For each of the examples below, please answer the following:

1. Does an externality exist? If so, classify the externality as positive/negative (or both).
2. If an externality exists, determine whether the Coase theorem applies (i.e. is it possible/reasonably feasible to asign property rights and solve the problem?)
3. If an extenality exists and the Coase theorem does not apply, argue which of the government's tools are best suited to address the issue: quantity regulation, taxes/subsidies, tradeable permits, or something else.

Consider the following examples:

1. British Petroleum drills for oil in the gulf coast
   - Yes; accidents on oil rigs that cause spills impose a negative externality on others (e.g. inhabitants of gulf states). (optional answer: Oil drilling may also yield a positive externality if the identification of the location of oil allows other companies to drill for oil more effectively b/c they know where the oil is).
   - If oil spills only damage property, and these property owners can costlessly recoup costs in the legal system, then the drillers will internalize the impact of their drilling on the social cost of the oil spill. But, if it is hard to determine the true costs from an oil spill (e.g. may be hard to figure out whether someone lost their job b/c of an oil spill or b/c of some other reason), then the coase theorem may not apply. (Also, in the positive externality case: may be difficult to assign property rights to an oil field after it is identified, so coase theorem may not apply).

- Quantity regulation on the amount of safety/advanced drilling technology investment seems feasible. One could also argue for subsidies for safer drilling technologies (or taxes on less safe technologies). Tradeable permits seems difficult here, since it's not clear what the permits would specify (but I'm open to a good suggestion!).

2. Carbon emissions from vehicles

   - Yes; Classic negative externality: I drive my car and emit gases that harm others (whose harm I don't pay for).
   - Coase theorem is unlikely to apply in this case, since it would require assigning property rights to those that are harmed. Since many of the harmed are very dispersed (e.g. driving in boston theoretically harms everyone in the world a small amount) and in some cases involves the "unborn" (future generations facing global warming), the feasibility of negotiated private contracts is highly questionable.
   - As discussed in class, if we believe that the benefit curve is flat, we would want to price the carbon using a tax. Tradeable permits may be more desirable politically though. Quantity regulation would require differential quantities for each producer of carbon, since they all have differing marginal costs; therefore quantity regulation seems suboptimal/difficult without instituting tradeable permits.

3. Your upstairs neighbors throwing an awesome, but loud party

   - Yes; but the externality is either positive or negative, depending on your taste for parties.
   - Coase theorem would require the neighbors to own the rights to holding the party. Then the neighbor would pay the other neighbor to have (or not have) the party. This could work (so an answer of "yes" is fine). But, in reality, there are likely many different people who are affected by the throwing of the party (e.g. multiple neighbors hate the noise). Bargaining with all parties may allow one party to "hold-up" the others, rendering the coase theorem inapplicable.

4. Buying a car with added safety features that prevent the drivers/passengers' deaths in the event of an accident

   - Depends; If people drive more recklessly as a result of having a safer car, then buying the safety feature imposes a negative externality on other drivers. If having a safety feature does not change the likelihood of an accident or the impact on the other cars, then there is no externality.
   - The coase theorem does not apply: It would be incredibly difficult to write a contract with those with whom you may eventually be engaged in a car accident.

- Quantity regulation (e.g. regulating the safety feature, or preventing it), or taxation would correct the externality (Yes, that's right, we'd theoretically want to tax the safety feature if it causes people to drive more recklessly).

5. Bringing crying babies on a plane

   - Yes; this is the worst negative externality known to all of mankind.
   - Coase theorem does not apply: have you ever tried reasoning with a crying baby?
   - Not many good solutions here. Sure, you can tax parents that bring babies on the plane and rebate the tax to those that are exposed to the crying. The bigger question though is why don't airlines already do more of this and lower the ticket price to everyone who is subjected to a crying baby (or just serve free drinks or something when a baby starts crying)? It would seem that the airlines are in a better position to solve this than is the government. Why don't airlines do this?

2 Question Two

An natural gas company in San Francisco owns many pipelines running underneath what is now populated areas. The company can invest $\$u$ in the maintenance of the pipes. Maintenance affects two things. First, more maintanence means that the gas company will lose less gas in the pipes. Assume that the value of lost gas is given by $\frac{1}{u}$ so that more maintenance reduces the amount of lost gas. Second, more maintenance means less damage to the land above the pipes. Assume that value of the damage to the land above the pipes is given by $3 \cdot \frac{1}{u}$, so that more maintenance decreases the amount of damage to the land above.

1. What is the socially optimal level of maintenance, u? What is the value of lost gas? What is the value of land damage?

   - The social optimum minimizes total costs:

   $$\min u + 3\frac{1}{u} + \frac{1}{u}$$

   or

   $$1 = 4\frac{1}{u^2} = 0$$

   or

   $$u^p = 2$$

   The socially optimal amount of maintenance is $u^s = 2$. The value of lost gas is $\frac{1}{2}$ and the value of land damage is $\frac{3}{2}$.

2. What level of u is chosen by the gas company when no one owns the land above the pipes? Now what is the value of lost gas? What is the value of land damage? What is the deadweight loss?

   - The gas company will solve

   $$\min u + \frac{1}{u}$$

   so that

   $$\frac{1}{(u^p)^2} = 1$$

   or $u^p = 1$. The value of lost gas is 1 and the value of land damage is 3. The total social costs are therefore $1+1+3 = 5$. In the social optimum, the social costs are $2+\frac{1}{2}+\frac{3}{2} = 4$. Therefore, the deadweight loss is $5 - 4 = 1$.

3. Suppose now that the gas company owns the land above the pipes. What level of u will they choose now? Is this optimal? If not, calculate the deadweight loss.

   - Gas company will mimize costs that include the damage to the land:

   $$\min u + 3\frac{1}{u} + \frac{1}{u}$$

   and choose $u^p = u^s = 2$, which is optimal. There is no deadweight loss.

4. Suppose now that Jimmy Fallon, an ordinary private citizen, owns the property above the plant and can costlessly sue the natural gas company for the losses to his property. What level of u will be chosen by the natural gas company? How much will be paid from the gas company to Jimmy Fallon?

   - Jimmy Fallon's lawsuits impose a cost on the gas company of $P(u) = 3\frac{1}{u}$. They will take this into account in their choice of u, choosing to minimize:

   $$\begin{aligned} & \min u + \frac{1}{u} + P(u) \\ = & \min u + \frac{1}{u} + 3\frac{1}{u} \end{aligned}$$

   Therefore, they choose $u = 2$, the social optimum. The gas company will pay Jimmy $\frac{3}{2}$ for his property damage.

5. Suppose now that the courts are imperfect: For every \$1 in actual damage, only 50% of the damage can be recouped in court. So, if the true damage to Jimmy is L, the gas company will only pay $\frac{L}{2}$.

   (a) Suppose Jimmy Fallon owns the property. What level of u will be chosen by the gas company? Is this efficient? If not, what is the deadweight loss?

- In this case, the gas company only pays $P(u) = \frac{1}{2}\left(3\frac{1}{u}\right)$. Therefore, they minimize

$$\min u + \frac{1}{u} + \frac{1}{2}\left(3\frac{1}{u}\right)$$

or

$$1 - \frac{5}{2}\frac{1}{u^2} = 0$$

or

$$u = \sqrt{\frac{5}{2}}$$

This is not efficient ($u < 2$). The total social costs are now

$$\sqrt{\frac{5}{2}} + 4\frac{1}{\sqrt{\frac{5}{2}}}$$

so that the deadweight loss is

$$4 - \left(\sqrt{\frac{5}{2}} + 4\frac{1}{\sqrt{\frac{5}{2}}}\right)$$

(b) Suppose the gas company owns the property. What level of u will be chosen? Is this efficient? If not, what is the deadweight loss? If your answer is different than in (a), why? Have we violated an assumption of the coase theorem?

- This is the same as in part (3). The solution is efficient. The 50% recoup rate imposes a transactions cost, which violates the assumptions of the coase theorem.

Question Three

Two power plants provide power to all of Cambridge: an MIT plant and a Harvard plant. Both power plants burn coal to produce electricity, and consequently produce smog as a by-product. The MIT power plant could reduce its smog, but at a total cost:

$$c_M(x_M) = 5 \cdot x_M^2$$

where x_M indicates the total number of units of smog abated by MIT. The Harvard plant is slightly less efficient, and its total cost for cutting down on smog by x_H is:

$$c_H(x_H) = 7 \cdot x_H^2 + 10 \cdot x_H.$$

The Cambridge government hires a team of environmentalists who calculate that the total benefit of smog abatement to the city of Cambridge is $100 \cdot (x_M + x_H)$.

1. Calculate the socially optimal level of abatement for each power plant.

   - The social optimum equates marginal cost to marginal benefit:

$$\begin{aligned} 10x_M &= 100 \implies x_M = 10 \\ 14x_H + 10 &= 100 \implies x_H = \frac{90}{14} \end{aligned}$$

2. The Cambridge government considers imposing a tax on power production.

   (a) What tax should it impose to reach the abatement amounts you calculated in part (1)?[1]

   - The optimal tax would be the negative of the marginal benefit, $\tau = -100$ per unit of "non-abatement", or alternatively a subsidy of 100 for each unit abated.

   (b) Write down each firm's optimization problem under the tax, and show that each will privately choose the socially optimal abatement amount.

   - Each firm, $i = H, M$, chooses to maximize

$$\max 100x_i - c_i(x_i)$$

   so that

$$100 = c_i'(x_i) = 10x_M = 14x_H + 10$$

   and each firm chooses optimally.

3. Suppose that instead of taxation, the Cambridge government tries to regulate quantities. However, the city of Cambridge cannot write a law for each firm, so it simply declares that all Cambridge power plants must cut down on smog by $x_C \equiv 1$ units each year. Show that this is not efficient with BOTH math and intuition.

   - Intuitively, this is inefficient because the marginal cost of abatement is different across the plants. The plants would like to abate at different levels. Mathematically, the marginal cost of abatement at Harvard is $c_H'(1) = 14 + 10 = 24$. But, the marginal cost of abatement at MIT is $c_M'(1) = 10$. Therefore, if MIT abated a bit more and Harvard a bit less, there could be more abatement for less cost.

4. Suddenly, an economist is voted in as Mayor of Cambridge. She declares that Cambridge power plants must cut down on smog by 5 units *overall*. Additionally, she declares that firms will be able to competitively trade permits that will allow them NOT to abate. One of the

[1] Hint: we can think of a Pigouvian tax here as a subsidy on abatement. So taxes on pollution provide firms an incentive to abate.

mayor's old classmates from graduate school runs the MIT power plant, so the Mayor grants MIT 5 permits and Harvard 0 permits. As a result, Harvard is expected to abate by 5 units, and MIT (since it owns all the permits) is not expected to abate at all.

(a) Harvard will surely want to buy some of MIT's permits. Explain intuitively (no math), why this trade might happen.

- The marginal cost of abatement at harvard when $x_H = 0$ is 10, while the marginal cost of abatement at MIT is $10 * 5 = 50$. It's a lot cheaper to abate at Harvard than at MIT at the point where harvard does $x_H = 0$ and MIT does $x_M = 5$.

(b) Denote the number of permits that MIT holds as y_M (so that $x_M = 5 - y_M$), and denote the competitive price of permits as p. Derive the amount of permits that MIT will eventually hold as a function of p.

- MIT will choose to purchase permits until

$$p = c'_M(x_M) = 10x_M$$

so that

$$x_M = \frac{p}{10}$$

or

$$y_M = 5 - \frac{p}{10}$$

is the demand for permits.

(c) Calculate the amount of permits that Harvard will hold as a function of p.

- Harvard will purchase permits until

$$p = c'_H(x_H) = 14x_H + 10$$

or

$$x_H = \frac{p - 10}{14}$$

or

$$y_H = 5 - \frac{p - 10}{14}$$

(d) Using that fact that $y_M + y_H = 5$, calculate p.

- We have

$$\begin{aligned} 5 - \frac{p - 10}{14} + 5 - \frac{p}{10} &= 5 \\ 5 &= \frac{p}{10} + \frac{p - 10}{14} \end{aligned}$$

or

$$p\left(\frac{1}{10}+\frac{1}{14}\right)=5+\frac{10}{14}$$

$$p=\frac{5+\frac{10}{14}}{\frac{1}{10}+\frac{1}{14}}=\frac{100}{3}$$

(e) If the new mayor had divided the permits up differently, what outcomes would have changed and what would have stayed the same?

- The abatement by each firm would remain the same. The equilibrium abatement levels do not depend on who owned the permits in the first place. But, the profits for each company depend on who owns the permits in the first place. If MIT owns the permits, they can sell them to Harvard and make more money (and vice-versa). But the amount of abatement done at each plant does not depend on who owned the permits in the first place.

Question Four

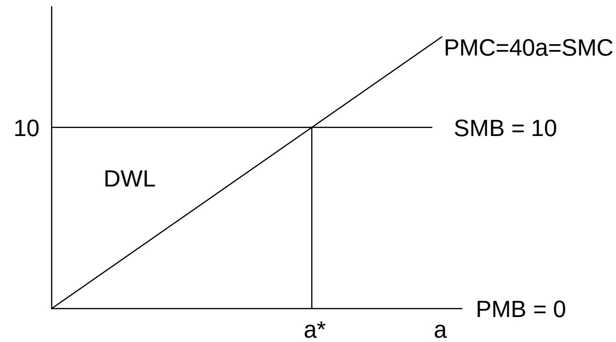
Vermont Hardwood crafts solid wood furniture using a combination of time-tested hand construction and modern finishing techniques. Residual wood finishing chemicals are washed away as run-off and deposited in the nearby lake, a favorite fishing site for locals. A variety of technologies, including high volume, low pressure sprayers and on-site solvent recovery sills are available for implementation. These technologies allow the manufacturer to reduce chemical emissions at a cost:

$$C_1(a)=20\cdot a^2$$

where a is the level of pollution abatement. A city planner determines that the benefit to the residents of pollution abatement is 10 per unit.

1. Sketch a graph depicting the private marginal costs and benefits of abatement, and label the private market equilibrium. On the same set of axis, sketch the social marginal costs and benefits of abatement, and label the efficient outcome. Indicate the DWL if the city takes no action.

- The private marginal benefit is zero. The private marginal cost is $mc = 40a$.



2. Calculate the level of pollution abatement that is socially efficient.

   - SMC=SMB when $40a = 10$, or $a = \frac{1}{4}$.

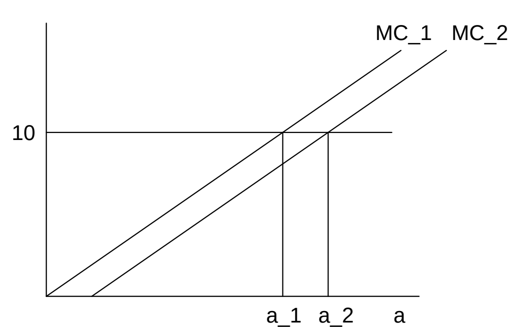
3. If the city institutes a per-unit tax on chemical emissions, what specific tax (τ^*) will reach the socially optimal amount of abatement?

   - The city should choose $\tau^* = 10$.

   The city planner is considering either taxing the firm's pollution or requiring the firm to reach a minimum level of pollution abatement. However, given constant progress in abatement technologies the costs of abatement *might* reduce to: $C_2(a) = 20 \cdot a^2 - a$. Thus while the social benefits of abatement are known, the social costs are uncertain.

4. Suppose that the planner institutes the per-unit tax calculated in (b). Assume that the true costs of abatement are revealed as $C_2(a) = 20 \cdot a^2 - a$. Illustrate the problem graphically and indicate the DWL relative to the social optimum. What level of abatement will be undertaken by the firm? Calculate the DWL.

   - We have $MC_2 = 40a_2 - 1$. Now, the firm will choose $40a_2 - 1 = 10$, or $a_2 = \frac{11}{40}$. There is no deadweight loss. The firms adjust to the social optimum in response to the taxation.



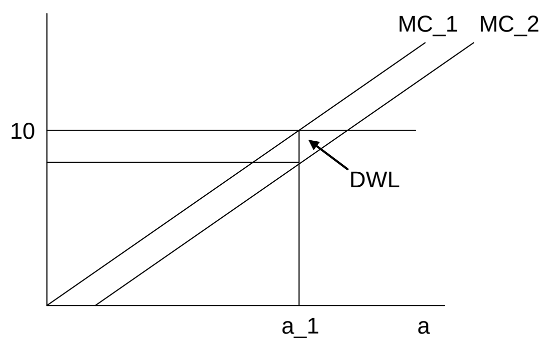
5. Suppose instead that the planner institutes a mandatory minimum abatement at the socially optimal level found in (2). Again, assume that the true costs of abatement are revealed as

$C_2(a) = 20 \cdot a^2 - a$. Illustrate the problem graphically and indicate the DWL relative to the social optimum. What level of abatement will be undertaken by the firm? Calculate the DWL.

- In this case, the firm abates $a = \frac{1}{4}$ when the socially optimal level of abatement is $\frac{11}{40}$. Total welfare under the socially optimal case is $\frac{11}{40}9 - 20\left(\frac{11}{40}\right)^2$. Total welfare when abatement is $\frac{1}{4}$ is $\frac{1}{4}9 - 20\left(\frac{11}{40}\right)^2$, yielding a DWL of

$$DWL = \frac{11}{40}9 - 20\left(\frac{11}{40}\right)^2 - \left(\frac{1}{4}9 - 20\left(\frac{11}{40}\right)^2\right)$$

  as shown in the graph:



6. Given the uncertainty in abatement costs, which strategy makes the most sense for reducing pollution in this context?

- Taxation is better than quantity mandates is better

7. Intuitively discuss what is driving this result.

- Price controls are better than quantity regulation when the SMB curve is relatively flat (relative to the SMC curve). Setting a "price" allows one to get "closer" to the true social benefit when the social benefit curve is flat. If it's steep, then using quantity regulation allows one to get close to the true social benefit. In some sense, a flat SMB curve allows one to be relatively more confident in the level of the tax as opposed to the quantity of the abatement.

Question Five

Gilroy, CA is the garlic capital of the world. Unfortunately, the stench of garlic permeates all aspects of life in the city. There are only two residents willing to live within city-limits, Abe and Betty. Abe earns an income of 460, and Betty earns an income of 440. A traveling salesman is visiting the town, offering odor conversion units which conveniently inputs garlic odor and outputs clean air. Preferences over clean air (C) and all private consumption goods (x_i) for individual i are given by:

$$U_i = 5 \cdot ln(x_i) + ln(C)$$

The total provision of clear air is given as the sum of individual purchases: $C = C_A + C_B$ ($+C_G$ when the local government purchases clean air in parts (4)-(5)). The price of clean air is 2 while the price of all other consumption goods is 1.

1. For both Abe and Betty, calculate each individual's private provision of clean air, taking the other's provision as given. That is, solve for C_A as a function of C_B in Abe's optimization problem (and solve for C_B as a function of C_A in Betty's optimization problem). Can you explain the sign on the contribution of the other resident in these response functions?

   - Let y_A, y_B be the income of Abe and Betty. Given C_B, Abe's maximization problem is

     $$\max_{x_A, C_A} 5\ln(x_A) + \ln(C_A + C_B)$$
     $$st \;\; x_A + 2C_A \leq y_A$$

     so, if λ_A is the lagrange multiplier on the BC, we have

     $$\begin{aligned}[x_A] &: \frac{5}{x_A} = \lambda \\ [C_A] &: \frac{1}{C_A + C_B} = 2\lambda\end{aligned}$$

     and

     $$x_A = y_A - 2C_A$$

     so that

     $$\lambda = \frac{5}{y_A - 2C_A}$$

     and so

     $$\frac{1}{C_A + C_B} = \frac{10}{y_A - 2C_A}$$

     or

     $$\begin{aligned}10(C_A + C_B) &= y_A - 2C_A \\ 12C_A &= y_A - 10C_B \\ C_A &= \frac{y_A}{12} - \frac{5}{6}C_B \\ C_A &= \frac{460}{12} - \frac{5}{6}C_B\end{aligned}$$

     Now, also, by reversing A and B and noticing the convenient symmetry of the maximization problem, we have

$$
\begin{aligned}
C_B &= \frac{y_B}{12} - \frac{5}{6}C_A \\
C_B &= \frac{440}{12} - \frac{5}{6}C_A
\end{aligned}
$$

- Note that an individual's contribution is inversely related to the contribution of others - this is because it's a public good! If the other person contributes to air quality, it decreases my own marginal benefit to contributing to air quality, so I invest less in it and choose to consume more x.

2. If the government does not intervene, what level of clean air will be provided? How many units are provided by Abe? How many by Betty?

    - We search for the levels C_B and C_A such that both equations hold:

$$
\begin{aligned}
C_B &= \frac{440}{12} - \frac{5}{6}C_A \\
&= \frac{440}{12} - \frac{5}{6}\left(\frac{460}{12} - \frac{5}{6}C_B\right) \\
C_B\left(1 - \frac{5}{6}\frac{5}{6}\right) &= \frac{440}{12} - \frac{5}{6}\frac{460}{12} \\
C_B &= \frac{\frac{440}{12} - \frac{5}{6}\frac{460}{12}}{1 - \frac{5}{6}\frac{5}{6}} = \frac{170}{11}
\end{aligned}
$$

and

$$
\begin{aligned}
C_A &= \frac{460}{12} - \frac{5}{6}C_B \\
C_A &= \frac{460}{12} - \frac{5}{6}\frac{170}{11} = \frac{280}{11}
\end{aligned}
$$

so that

$$
C = \frac{170}{11} + \frac{280}{11} = \frac{450}{11}
$$

3. What is the socially optimal level of clean air provision? (You may assume a utilitarian social welfare function) Does this value differ from that found in (2)? Explain in the context of externalities.

    - The social optimum is given by the samuelson condition

$$
MRS_A + MRS_B = priceratio
$$

or

$$
\frac{\frac{1}{C}}{\frac{5}{x_A}} + \frac{\frac{1}{C}}{\frac{5}{x_B}} = \frac{2}{1}
$$

or

$$\begin{aligned} \frac{x_A}{5C} + \frac{x_B}{5C} &= 2 \\ x_A + x_B &= 10C \end{aligned}$$

using the pooled budget constraint, we have

$$x_A + x_B + 2C = 460 + 440 = 900$$

so

$$x_A + x_B = 10C = 900 - 2C$$

so

$$12C = 900$$

or

$$C^* = \frac{900}{12} > \frac{450}{11}$$

4. Suppose the local government is dissatisfied with the level of private provision. The government taxes both Abe and Betty 30 each in lump-sum fashion (net-of-tax incomes are effectively reduced to 440 and 410 respectively) to provide 30 units of clean air. Both Abe and Betty are free to purchase additional units of clean air if they find it privately optimal to do so. What is the total level of clean air provided? Clearly explain the impact of the taxation/provision by the local government on the private provision by each resident. How does this answer compare to (2)?

   - Now, abe will choose

$$\begin{aligned} C_A &= \frac{y_A}{12} - \frac{5}{6}(C_B + 30) \\ &= \frac{460 - 30}{12} - \frac{5}{6}(C_B + 30) \\ &= \frac{430}{12} - \frac{5}{6}30 - \frac{5}{6}C_B \end{aligned}$$

   and Betty will choose

$$\begin{aligned} C_B &= \frac{440 - 30}{12} - \frac{5}{6}(C_A + 30) \\ &= \frac{410}{12} - \frac{5}{6}30 - \frac{5}{6}C_A \end{aligned}$$

so equilibrium will be determined by

$$\begin{aligned}
C_B &= \frac{410}{12} - \frac{5}{6}30 - \frac{5}{6}C_A \\
C_B &= \frac{410}{12} - \frac{5}{6}30 - \frac{5}{6}\left(\frac{430}{12} - \frac{5}{6}30 - \frac{5}{6}C_B\right) \\
C_B &= \frac{410}{12} - \frac{5}{6}\frac{430}{12} - \frac{5}{6}\left(1 - \frac{5}{6}\right)30 + \frac{5}{6}\frac{5}{6}C_B \\
C_B\left(1 - \frac{5}{6}\frac{5}{6}\right) &= \frac{410}{12} - \frac{5}{6}\frac{430}{12} - \frac{5}{6}\left(1 - \frac{5}{6}\right)30 \\
C_B &= \frac{\frac{410}{12} - \frac{5}{6}\frac{430}{12} - \frac{5}{6}\left(1 - \frac{5}{6}\right)30}{1 - \frac{5}{6}\frac{5}{6}} = \frac{5}{11} < \frac{170}{11}
\end{aligned}$$

and so

$$\begin{aligned}
C_A &= \frac{430}{12} - \frac{5}{6}30 - \frac{5}{6}C_B \\
&= \frac{430}{12} - \frac{5}{6}30 - \frac{5}{6}\frac{5}{11} = \frac{115}{11} < \frac{280}{11}
\end{aligned}$$

So, the total amount of clean air is

$$C = 30 + \frac{115}{11} + \frac{5}{11} = \frac{450}{11}$$

Which is the same as in part (2)! The government provision of clean air completely "crowds-out" the clean air that would be provided by the agents.