14.382 MIDTERM 2006

Answer as if your try to explain the material to your fellow student.

Consider the model, where $Y = X\beta + \epsilon$, where for each t, $\epsilon_t \sim \sigma(e_t - 1)$, where e_t is standard exponential variable such that $E[e_t] = 1$ and $Var[e_t] = 1$. Assume that X are independent of ϵ. Suppose that (x_t, ϵ_t) are i.i.d. across t.

1. (10) Do Gauss-Markov assumptions hold for this model?

2. (10) Consider the least squares estimator $\hat{\beta}$. Compute $E[\hat{\beta}|X]$ and $Var[\hat{\beta}|X]$. Is $\hat{\beta}$ normally distributed in finite samples, conditional on X?

3. (10) Carefully, but briefly, explain the label "BLUE". Is OLS BLUE in this set-up?

4. (10) Consider estimating the following effect

$$E[y_t|x_t = x''] - E[y_t|x_t = x'] = (x'' - x')'\beta$$

Give an economic example where such an effect might be of interest. Is $(x'' - x')'\hat{\beta}$ BLUE for this effect? Why or why not?

5. (10) Is OLS the BUE (best unbiased estimator) in this model? A brief answer suffices.

6. (15) What is the large sample distribution of $\hat{\beta}$? Make any additional primitive assumptions you might need. [Note: high level assumptions will receive partial credit.]

7. (10) Construct a consistent estimator for the large sample variance of $\hat{\beta}$. Prove its consistency by making any additional assumptions you need.

8. (10) Suppose we want to test the null hypothesis $H_0 : \beta_j = 0$ vs $H_A : \beta_j < 0$. Construct a t-statistic for testing this hypothesis. Derive its limit distribution and describe how to select critical value for this test to maintain the level of significance equal to 5%.

9. (15) Suppose the sample size $n = 6$. Do you expect the large sample distribution to be a good approximation to the exact distribution of the t-statistic in question 8. Discuss how to get the exact distribution of the t-statistic. How would you generate p-values (or critical values) for checking the hypothesis of question 8 that would be valid even for $n = 6$?

10. (Extra Points) Can you come up with better estimators than OLS for this model? Hint: think about the trivial case first, where $X_t = 1$.