14.381: Statistics

Problem Set 6

1. Let $X_1, \ldots, X_n$ be iid with

$$f(x \mid \theta) = \begin{cases} e^{-(x-\theta)}; & \theta \leq x < \infty \\ 0; & \text{otherwise} \end{cases}$$

Let us define $Y = \min\{X_1, \ldots, X_n\}$.

(a) Find the pdf $f_Y(y \mid \theta)$ of Y.

(b) For each realization of $Y = y$ find values of $\theta_U(y)$ and $\theta_L(y)$ such that

$$F_Y(y \mid \theta_U(y)) = \frac{\alpha}{2}; \quad F_Y(y \mid \theta_L(y)) = 1 - \frac{\alpha}{2}$$

(c) Show that $[\theta_L(Y), \theta_U(Y)]$ is a confidence interval for θ with a confidence level equal to $1 - \alpha$.

2. Suppose that $X_1, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$ with known σ^2. Find a minimum value of n to guarantee that a 0.95 confidence interval for μ will have length no more than $\frac{\sigma}{4}$.

3. Assume that $X_1, \ldots, X_n$ are iid Poisson (λ)

(a) Construct a Wald type confience set for λ.

(b) Construct a confidence set for λ by inverting Lagrange multiplier (score) test.

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