14.381: Statistics

Problem Set 5

1. Let $X_1, \ldots, X_n$ be iid Poisson (λ).

   (a) Find the UMP test for $H_0 : \lambda \leq \lambda_0$ vs. $H_1 : \lambda > \lambda_0$

   (b) Consider the specific case $H_0 : \lambda_1 \leq 1$ vs. $H_1 : \lambda > 1$. Determine the sample size n so that the UMP satisfies two conditions:

   $$P_{\lambda=1}(\text{reject } H_0) \approx 0.05$$
   $$P_{\lambda=2}(\text{reject } H_0) \approx 0.9$$

   Here "$\approx$" stays for " approximately equal". Please, use the CLT as approximation device.

2. Suppose that we have two independent samples: $X_1, \ldots, X_n$ are iid exponential(θ) and $Y_1, \ldots, Y_m$ are iid exponential(μ). Both θ and μ are unknown. We want to test $H_0 : \mu = \theta$ vs $H_1 : \mu \neq \theta$. The goal of this problem is to write down a LR test statistic.

   (a) Write down the likelihood function and find the unrestricted ML estimates of μ and θ

   (b) Find the restricted ML estimate (via imposing the null)

   (c) Write down LR test statistic

   (d) Show that it's a function of test statistic $\frac{\sum_i X_i}{\sum_i X_i + \sum_j Y_j}$

3. Let $X_1, X_2, \ldots, X_N$ be a sequence of independent and identically distributed random variables with common probability function

   $$f_X(x \mid \theta_0) = \frac{\theta_0(-\ln \theta_0)^x}{x!}$$

   for $x = 0, 1, 2, \ldots$ for some unknown $0 < \theta_0 < 1$.

(a) Find the maximum likelihood estimator for θ_0.

(b) Suppose that $N = 100$, $\sum_{i=1}^{N} x_i = 500$. Test the hypothesis that $\theta_0 = e^{-4}$ against the hypothesis that $\theta_0 \neq e^{-4}$ at the 5% level using a likelihood ratio test.

(c) Test the hypothesis that $\theta_0 = e^{-4}$ against the hypothesis that $\theta_0 \neq e^{-4}$ at the 5% level using Rao's score test (also known as a Lagrange multiplier test).

(d) Test the hypothesis that $\theta_0 = e^{-4}$ against the hypothesis that $\theta_0 \neq e^{-4}$ at the 5% level using a Wald test.

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