

**14.20 - Problem Set 2**  
**Due in-class (1 p.m.) on Wednesday, March 12**

1. Suppose the market demand for lumber were given by:

$$P(Q) = 100 - Q/2$$

There are 2 symmetric producers in the market, each with the following cost function:

$$C(q) = 10q$$

- a. What would be the monopoly price, quantity, and profits in this market?
- b. Suppose the two firms compete in the following supergame:
  - (i) Each firm produces  $q_i = q^*$
  - (ii) If any firm produces  $q > q^*$ , then each firm believes that both will revert to the one-shot Cournot quantity,  $q^{ct}$ , forever.

What is the critical value of the firms' common discount factor,  $\delta$ , such that  $q^* = .5Q^m$ , where  $Q^m =$  joint monopoly output, is the equilibrium outcome to this game?

- c. Suppose the detection lag (the time required to discover and respond to cheating) were 2 periods? How would this change the critical value of  $\delta$ ?
- d. Suppose the firms instead set price, given the cost functions above and no capacity constraints. What is the equilibrium price and quantity to this stage (one-shot) game?
- e. Let the firms in part (d) compete repeatedly in the following Bertrand supergame:
  - (i) Each firm sets  $p_i = p^*$
  - (ii) If any firm charges  $p < p^*$ , then each firm believes that both firms will revert to the one-shot Bertrand price,  $p^B$ , forever.

What is the critical value of the discount factor,  $\delta$ , such that  $p^* = p^M$  is the equilibrium outcome? Which type of competition, price or quantities, is more likely to sustain the monopoly outcome? Why?

- f. Suppose the firms compete in prices, but are capacity-constrained. Firm 1 has capacity to produce up to 140 units; firm 2 has capacity sufficient to produce up to 80 units. Describe (qualitatively) how you expect this to affect the equilibrium.

2. Consider a homogeneous product market that looks much like the market in (1), except that there are 5 identical firms in the market. Let the market demand be given by:

$$P(Q) = 100 - Q/2$$

Each firm produces with a cost function given by:

$$C(q_i) = 10q_i \quad i = 1, \dots, 5$$

The CEOs of the five firms run into each other at a dinner during the World Economic Forum in Davos, Switzerland. They agree to form a cartel. While the firms agree to make joint output decisions, they cannot make side payments among themselves.

- a. What production quota should the cartel set to maximize the joint profits of the five firms? What will be the output of each firm, the price of the product, and the profits of each firm?
  - b. You have been asked to evaluate whether this cartel is likely to be stable. Assume a three-stage game. First, firms decide whether to participate in the cartel. Second, the cartel, knowing who is in or out, commits to a production quota. Third, any non-cartel firms choose their output to maximize profits, given the cartel production. In this variant of the one-shot Stackelberg game, with the cartel as the first-mover, would all five firms agree to participate in the cartel?
  - c. Is there any size of cartel that is stable in this market (i.e., those in the cartel do not want to defect, and those outside the cartel do not want to join)? Can you give an intuition for this result?
3. The demand for two brands of laundry detergent, Wave (W) and Rah (R), are given by the following demands:

$$Q_W = 80 - 2P_W + P_R$$

$$Q_R = 80 - 2P_R + P_W$$

The firms have identical cost functions,  $C(Q_i) = 10Q_i$   $i = W, R$

Suppose the firms compete in prices.

- a. What is the best response function for each firm (express firm  $i$ 's optimal price as a function of firm  $j$ 's price)? What is the equilibrium to the one-shot pricing game? What are the profits of each firm?
- b. Suppose the manufacturer of Wave could commit to setting  $P_W$  before the manufacturer of Rah could set  $P_R$ . How would this change the equilibrium? What are the profits of

each firm in this case? Should Wave take advantage of this commitment possibility? Why or why not?

4. Dynamic Games: Do problem 9.6 in Church and Ware (p. 301-302).
5. Consider a classic Bertrand trigger pricing supergame (e.g., 1(e) above). Describe how each of the following conditions would affect the sustainability of a collusive outcome, if at all. Be as specific as possible in your explanation.
  - a. The government's Competition Commission announces plans to publish a monthly list of all transactions prices and volumes in this market, in an effort to improve "market transparency" for consumers.
  - b. Recent regulations require users of the product to convert to less environmentally-hazardous substitutes over the next 5 years. At that point, production and sales of this product will be banned.
  - c. Demand for the product is highly variable, and individual firms cannot observe other firms' prices or sales.
  - d. The firms compete in several other product markets; some of which favor firm 1, some favor firm 2.
6. Read the assigned article, "How to Beat Priceline: New Sites Post Secret Bids ---Bargain Hunters Can See Exactly What Winners Have Paid for Cheap Trips," by Ron Lieber, *Wall Street Journal*, April 9, 2002 (available on the course website).
  - a. An identical hotel room appears to sell for one price if booked through the hotel and a different, lower price if booked through Priceline. What may account for the difference in the two prices (more than one factor may be potentially relevant)? Be as specific and complete as possible.
  - b. Describe any differences between the hotel and Priceline in their incentives for preventing customers from posting the price they paid on Priceline for a room in a specific hotel.