14.13 Economics and Psychology
MIT Spring 04

Problem Set #3

1 Bounded Rationality as Noise

1. Define $M_n = \max_{i=1...n} \epsilon_i$, where ϵ_i are iid with a cdf F and pdf f. Define $\overline{F}(x) = 1 - F(x)$.The intuition of Lemma 1 in lecture 9 is based on

$$\mathrm{E}[\overline{F}(M_n)] = \frac{1}{n+1} \tag{1}$$

   (a) [2 points] Call g_n the pdf of M_n and G_n the cdf. Show that

$$\begin{aligned} G_n(x) &= F(x)^n \\ g_n(x) &= nf(x)F(x)^{n-1} \end{aligned}$$

   (b) [3 points] Prove (1)

2. When the noise follows a Gumbel distribution, the demand has a closed form solution. It has been derived in lecture 8:

$$D_i(p_i, p_{-i}) = \frac{e^{\frac{q_i - p_i}{\sigma}}}{\sum_{j=1}^{n} e^{\frac{q_j - p_j}{\sigma}}}$$

   where $p_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$.

   (a) [5 points] Assume all the qualities are the same, q. In a symmetric Bertrand equilibrium every firm will choose the same price p^*. By definition of an equilibrium, if all the other firms choose p^*, i.e. $p_{-i} = p_{-i}^* = (p^*, ..., p^*, p^*, ..., p^*)$, firm i will choose p^*. Prove that the equilibrium price p^*

$$p^* = c + \frac{n}{n-1}\sigma$$

   satisfies

$$p^* = \arg\max_{p_i} (p_i - c) D_i(p_i, p_{-i}^*)$$

   Hint: maximizing the profit is the same as maximizing ln of the profit.

   (b) Order the quantities $q_1 > q_2 > ... > q_n$, assume the prices are zero $p_i = 0$

      i. [3 points] Show that when the variance of the noise becomes very large ($\sigma \to +\infty$), the market shares become equal $D_i = \frac{1}{n}$

ii. [3 points] Show that when the variance of the noise becomes very small ($\sigma \to 0$), firm 1, the high quality firm, gets the whole demand $D_1 = 1$ and $D_i = 0$ for $i > 1$.

3. Consumers have a unit demand for a good, if they don't buy it they get 0. A consumer gets $q - p$ if he consumes the good. A sophisticated consumer is able to determine this value whereas a naive one gets only a signal of this value. The signal is $q - p + \sigma\epsilon$ where $\epsilon = \pm 1$ with probability .5. Let α be the proportion of naive consumers. The firm can choose both the price p and the noise σ. Assume that the cost of production is 0 and that the cost of choosing the degree of complexity of the product is $c(\sigma) = \frac{1}{2\gamma}(\sigma - \sigma^*)^2$. Assume $q > \sigma^* - \frac{\gamma}{2} > 0$.

   (a) [5 points] Give an example of how a firm could manipulate σ. Explain why the social optimal degree of complexity is not 0.

   (b) [7 points] What is the profit of this firm as a function of p and σ?

   (c) [12 points] What p and σ will the firm choose depending on how large q is?

   (d) [5 points] Assume α is close to 1, the population is mainly composed of naive consumers. Will a high/low quality firm choose an excessive complex/simple product? Interpret.

   (e) [5 points] Assume α is close to 0, the population is mainly composed of sophisticated consumers. Will a high/low quality firm choose an excessive complex/simple product? Interpret.

2 Shrouded Attributes: continuous add-on

- The utility of a consumer who buys the base good at price p and $\widehat{q}$ units of the add-on at price $\widehat{p}$ is:
$$V - p + u(\widehat{q}, e) - \widehat{p}\widehat{q}$$
where e represents a costly effort the consumer can take to decrease his marginal utility of consuming the add-on.

- Call $(p^*, \widehat{p}^*)$ the prices offered by the competitor firm.

- A naive consumer doesn't have enough foresight about the add-on, he makes no effort, chooses the product that maximizes $V - p$ and chooses the amount of add-on $\widehat{q}^N(\widehat{p})$ in order to maximize $u(\widehat{q}, 0) - \widehat{p}\widehat{q}$. Assume all the consumers are naive. When they choose between $V - p$ and $V - p^*$, they will buy at price p with probability $D(-p + p^*)$. The profit function of a firm offering $(p, \widehat{p})$ is:
$$\Pi(p, \widehat{p} \ / \ p^*, \widehat{p}^*) = \left(p - c + (\widehat{p} - \widehat{c})\widehat{q}^N(\widehat{p})\right) D(-p + p^*)$$

- Note c the cost of production of the base good and $\widehat{c}$ the unit cost of production of the add-on.

- Note $\mu = \frac{D(0)}{D'(0)}$

1. [7 points] Prove that the firm will charge the monopoly price for the add-on
$$\frac{\widehat{p} - \widehat{c}}{\widehat{p}} = \frac{1}{\eta^N}$$
where $\eta^N = -\frac{\widehat{p}\widehat{q}^{N'}(\widehat{p})}{\widehat{q}^N}$ is the elasticity of demand.

2. [8 points] Prove that in a symmetric equilibrium ($p^* = \arg\max_p \Pi(p, \widehat{p} \ / \ p^*, \widehat{p}^*)$), the base good is a loss leader
$$p^* - c = \mu - (\widehat{p} - \widehat{c})\widehat{q}^N(\widehat{p}) < \mu$$

3. [5 points] Give the intuition for those results.