1 Hidden information

- Screening: uninformed party makes offers.
  - informational rents;
  - adverse selection;
  - efficiency losses;
  - market failure.
- Signaling: informed party acts.
  - equilibrium selection;
  - separation and/or pooling;
  - (all of the above), "money burning";
  - market resurrection.

1.1 Akerlof's Lemons:

Sellers, buyers, (equal masses), $x \sim U[0, 10000]$, $s = x$, $b = \alpha x$, $\alpha > 1$.

Full information: Each car is sold, $p \in [x, \alpha x]$.

Incomplete info: Given p, all $x < p$ will be sold.

Average quality: $\frac{1}{2}p$, buyers will not buy if $\alpha < 2$.

1.2 One seller, one buyer

Buyer: 2 types, $\theta \in \{\theta_L, \theta_H\}$.

$u^B(\theta_i, q, T) = \theta_i v(q) - T$. $v(0) = 0$, $v' > 0$, $v'' < 0$.

Reservation Utility: $\bar{u}$.

Seller: $u^S = T - cq$. Probability of θ_L is β.

- Complete info:

Seller: $T_i - cq_i \rightarrow_{T_i, q_i}$ max, s.t. $\theta_i v(q_i) - T_i \geq \bar{u}$.

Solution: $u^B = \bar{u}$, $\theta_i v'(q_i) = c$.

Implementation: two-part tariff, a bundle, ...

- Incomplete info: Linear price:

Each buyer: $\theta_i v(q) - Pq \rightarrow$ max, $\theta_i v'(q_i) = P$.

Seller: (Monopoly) $(P-c)D(P) \rightarrow_P$ max.

$P_m = c - \frac{D(P)}{D'(P)}$.

Note: If both types are served, both have positive surplus.

- Single two-part tariff: (Z, P), where Z is fixed fee.

Can extract all surplus from type L, $Z \geq S_L(P)$.

If serving H market only: $P = c$, $Z = S_H(P)$.

If two: $S_L(P) + (P-c)D(P) \rightarrow_P$ max.

$P = c - \frac{D(P)+S'_L(P)}{D'(P)} = c - \frac{D(P)-D_L(P)}{D'(P)}$.

1.3 Optimal scheme

Obs1: only two contracts (T_i, q_i) (by revelation principle), 4 constraints: $2IC$ and $2IR$.

Obs 2: IRH not binding.

Obs 3: IRL not binding. (consider full information optimum).

$(\bar{u} = 0)$ $T_L = \theta_L v(q_L)$, $T_H = \theta_H v(q_H) - \theta_H v(q_L) + T_L$.

Seller's problem:

$\max_{q_L, q_H} \beta\, [\theta_L v(q_L) - cq_L] +$

$+ (1-\beta)\, [\theta_H v(q_H) - \theta_H v(q_L) + \theta_L v(q_L) - cq_H]$.

FOC (q_H): $\theta_H v'(q_H) = c$ — efficiency on top, $S_H > 0$.

FOC (q_L): $\theta_L v'(q_L) > c$ — under provision, $S_L = 0$.

1.4 Credit rationing

Invest 1, Return R with prob P.

Borrowers: $i = s, r$.

- A1: $p_i R_i = m > 1$,

  A2: $p_s > p_r$, $R_s < R_r$.

Lender: Has $1 > \alpha > \max\{\beta, 1-\beta\}$ to lend.

- Complete info: repay $D_i = R_i$.

- Incomplete info: (D as an instrument)

  $D = R_r$ (one type is served) or $D = R_s$ (credit rationing).?

- Random contract: (x_i, D_i), x_i is probability of financing.

  IRS and ICR are binding.

  $x_r = 1$, $x_s < 1$. No rationing. Safe borrowers are indifferent.

  A3: $p_s R_s > 1$, $p_r R_r < 1$.

Either both (cross-subsidy) or none are financed.

1.5 Multiple types: Finite N

$\theta_n > \theta_{n-1} > \ldots > \theta_1$, $n > 2$.

$u^B(\theta_i, q, T) = \theta_i v(q) - T$

- Seller: $\beta_i = \Pr(\theta_i)$.

$$\begin{cases} \sum_{i=1}^n \beta_i (T_i - cq_i) \to_{T_i, q_i} \max, \text{ s.t.} \\ IR : \forall i, \quad \theta_i v(q_i) - T_i \geq 0, \\ IC : \forall i, j \quad \theta_i v(q_i) - T_i \geq \theta_i v(q_j) - T_j. \end{cases}$$

- IR: binding only for θ_1.

- Spence-Mirlees single-crossing: $\frac{\partial}{\partial \theta}\left[-\frac{\partial u/\partial q}{\partial u/\partial T}\right] > 0$.

- IC: (1) Only local C matter;

ICij & ICji $\Rightarrow \left(\theta_i - \theta_j\right)\left[v(q_i) - v(q_j)\right] \geq 0 \Rightarrow q_i > q_j$.

$\theta_i v(q_i) - T_i \geq \theta_i v(q_{i-1}) - T_{i-1} \Rightarrow \theta_{i+1} v(q_i) - T_i \geq \theta_{i+1} v(q_{i-1}) - T_{i-1}$;

$\theta_{i+1} v(q_{i+1}) - T_{i+1} \geq \theta_{i+1} v(q_i) - T_i \geq \theta_{i+1} v(q_{i-1}) - T_{i-1}$.

- (2) Only Downstream C matter. All local DC bind.

Ignore upstream. If one local DC is loose, increase all upstream T.

- Solution: mimics two types.

  Express T_i, plug into objective function.

- Results: Efficiency on top, no surplus on the bottom.

1.6 Multiple types: Continuous support

- Seller: $\theta \sim F[0, \bar{\theta}]$.

$$\begin{cases} \int_0^{\bar{\theta}} \left[T_\theta - c q_\theta\right] f(\theta)\, d\theta \rightarrow_{T_\theta, q_\theta} \max, \text{ s.t.} \\ IR : \forall \theta, \quad \theta v(q_\theta) - T_\theta \geq 0, \\ IC : \forall \theta, \theta' \quad \theta v(q_\theta) - T_\theta \geq \theta v(q_{\theta'}) - T_{\theta'}. \end{cases}$$

- IR: only for $\theta = 0$ matters.

- IC: $W(\theta) \equiv \theta v(q_\theta) - T_\theta = \max_{\theta'} \left\{\theta v(q_{\theta'}) - T_{\theta'}\right\}$

- Thus, $\frac{dW(\theta)}{d\theta} = \frac{\partial W(\theta)}{\partial \theta} = v(q_\theta)$,

$W(\theta) = \int_0^\theta v(q_x) dx + W(0) = \int_0^\theta v(q_x) dx$.

$T_\theta = \theta v(q_\theta) - W(\theta)$.

- $\pi = \int_0^{\bar{\theta}} \left[\theta v(q_\theta) - \int_0^{\theta} v(q_x)dx - cq_\theta\right] f(\theta)d\theta.$

- $\pi = \int_0^{\bar{\theta}} L(q_\theta, T_\theta)f(\theta)d\theta \to \max_{q,T}$

- $L = \theta v(q_\theta) - cq_\theta - \frac{1-F(\theta)}{f(\theta)}v(q_\theta).$

- Thus, $\frac{\partial L}{\partial q} = 0$,

$$\left[\theta - \frac{1 - F(\theta)}{f(\theta)}\right] v'(q_\theta) = c.$$

- Results: Underconsumption for all $\theta < \bar{\theta}$.

- $p(q_\theta) \equiv T'_\theta = \theta v'(q_\theta)$,

  $\frac{p-c}{p} = \frac{1-F(\theta)}{\theta f(\theta)}.$

- Do not forget: $\frac{dq}{d\theta} \geq 0$.

2 Spence's Model

Worker's productivity: $r_H > r_L > 0$.

Firm's prior: $\beta_i = \Pr\{r = r_i\}$.

Education: $c_i(e) = \theta_i e$, $\theta_H < \theta_L$.

- Complete info: $e_L = e_H = 0$, $w_i = r_i$.

- Incomplete info:

σ_i — mixed strategy of i over e.

$\beta(r_i|e)$ — firm's posterior belief.

$w(e) = \beta(r_L|e)r_L + \beta(r_H|e)r_H$

- Solution: Perfect Bayesian Equilibrium

  PBE: $\{\sigma_H, \sigma_L, (\beta(r_i|e))_{e\in E}\}$, such that, for all i,

  1. $\forall e^* \in Supp\ \sigma_i$, $e^* \in \arg\max_e [w(e) - \theta_i e]$.
  2. $\beta(r_i|e) = \frac{\beta_i \Pr(\sigma_i = e)}{\sum_j \beta_j \Pr(\sigma_j = e)}$ whenever possible, otherwise not restricted.
  3. $w(e) = \beta(r_L|e)r_L + \beta(r_H|e)r_H$.

- Beliefs are restricted on-equilibrium path only.

- Three types of equilibria:

  1. Separating eqm: $e_H^* \neq e_L^*$.
  2. Pooling eqm: $e_H^* = e_L^*$.
  3. Semiseparating (mixed) eqm.

2.1 Analysis

- Separating equilibria:

$$S^S = \left\{ e_L^* = 0; \quad e_H^* \in \left[\frac{r_H - r_L}{\theta_L}, \frac{r_H - r_L}{\theta_H} \right] \right\}.$$

Beliefs: $\beta(r_H|e) = 1 \Leftrightarrow e \geq e_H^*$. $\beta(r_H|e) = 0$, otherwise.

- Pooling equilibria:

$$S^P = \left\{ e_L^* = e_H^* \in \left[0, \frac{\beta_L r_L + \beta_H r_H - r_L}{\theta_L} \right] \right\}.$$

Beliefs: $\beta(r_H|e) = \beta_H \Leftrightarrow e \geq e_H^*$. $\beta(r_H|e) = 0$, otherwise.

2.2 Refinements

Cho and Kreps' Intuitive criterion.

Suppose a deviator automatically reveals his type, will he still be willing to deviate?

Denote $u_i^* = w_i(e_i^*) - \theta_i e_i$. Consider $e \neq e_H^*, e_L^*$.

If $r_L - \theta_L e < u_L^*$ and $r_H - \theta_H e > u_H^*$, then $\beta(r_H|e) = 1$. (the other case similarly).

- Unique eqm: Least-Cost Separating eqm.

- Plausibility? $\beta_L \to 0$.

2.3 Maskin & Tirole Problem

Contract is offered before the signal is chosen: $\{w(e)\}$.

- Case 1 : $Er \leq r_H - \frac{\theta_H}{\theta_L}(r_H - r_L)$.

Unique eqm: $w(e)$ as in Least-Cost eqm.

- Case 2 : $Er \geq r_H - \frac{\theta_H}{\theta_L}(r_H - r_L)$. L-C can be improved.

2.4 Issues

- Competition:

  Auction for Lemon's: Can it work?

- Market design, regulation:

3 Hidden Action

- Pay before the service or after?
  - Before: Lousy Service.
  - After: Why pay?
- Sign a contract: Pay before or after?
- Moral hazard: Nobody's watching.
- References: B & D.
- (**) pay attention.

3.1 Simple model

- Agent: $u(w) - \psi(a) \rightarrow_a \text{max}$,

  $u' > 0$, $u'' \leq 0$, $\psi' > 0$, $\psi'' \geq 0$. $\psi(a) = a$.

- Principal: $V(q - w(q)) \rightarrow_{w(q\cdot),OB} \text{max}$
- (Not)Observable: Effort, Output, Noisy signal.

Technology: $q \in \{0, 1\}$, $Pr(q = 1|a) = p(a)$

$p' > 0$, $p'' < 0$, $p(0) = 0$, $p(\infty) = 1$, $p'(0) > 1$.

3.1.1 Everything observable (q, a)

$$PP = p(a)V(1-w_1)+(1-p(a))V(-w_0) \rightarrow_{a,w_0,w_1} \text{max}$$

s.t. $AG = p(a)u(w_1) + (1 - p(a))u(w_0) - a \geq \bar{u}$.

Solution: $\mathcal{L} = PP + \lambda AG \rightarrow$ max.

FOC:(w_0): $-(1-p(a))V'(-w_0)+\lambda(1-p(a))u'(w_o) = 0$

add (w_1): Borch rule (**)

$$\frac{V'(1-w_1)}{u'(w_1)} = \frac{V'(-w_0)}{u'(w_0)} = \lambda$$

FOC: (a): $p'(a)(V(1-w_1) - V(-w_0))$

$+\lambda(p'(a)(u(w_1) - u(w_0)) - 1) = 0$

- Ex 1:(**) $V(x) = x$. $u(w^*) = a^*$, $p'(a^*) = \frac{1}{u'(w^*)}$.

- Ex 2:(**) $u(x) = x$. $w_1^* - w_0^* = 1$, $p'(a^*) = 1$.

3.1.2 Only q is observable

Agent: $AG = p(a)u(w_1)+(1-p(a))u(w_0)-a \rightarrow_a$ max.

Principal: $PP(a, w_0, w_1) \rightarrow_{w_0,w_1}$ max.

s.t. $AG(a^*) \geq \bar{u}$, $a^* \in \arg\max_a AG(a)$.

FOC (AG): $p'(a)(u(w_1) - u(w_0)) = 0$.

- Ex 3. $V(x) = x$, $u(x) = x$, but $x \geq 0$.
  - First best: Sale of firm at price $-w_0^*$.
    If $w_0^* = 0$, $w_1^* = 1$, $AG(a) = p(a^*) - a^* > 0$. $PP = 0$.
  - Second-best: (**) Agent: $p'(a)w_1 = 1$.
    Principal: $p(a)(1 - w_1) \rightarrow_{w_1}$ max, s.t. above. Solution $\hat{a} < a^*$.

- general case: (**)

$$\frac{V'(1-w_1)}{u'(w_1)} = \lambda + \mu \frac{p'(a)}{p(a)}$$

$$\frac{V'(-w_0)}{u'(w_0)} = \lambda - \mu \frac{p'(a)}{1-p(a)}$$

For $\mu = 0$, Borch rule. But, often $\mu > 0$, so:

(**) reward for $q = 1$, punishment for $q = 0$.

Effort is known, reward-punishment create incentives.

3.2 Value of Information(?)

Observe, q, s, $s \in \{0, 1\}$, $Pr(q = i, s = j|a) = p_{ij}(a)$.

$$\frac{V'(i-w_{ij})}{u'(w_{ij})} = \lambda + \mu \frac{p'_{ij}(a)}{p_{ij}(a)}$$

When s goes away? When q is sufficient statistic) for a?

3.3 Continuous setting

- Principal $V(q-w)$. Agent: $u(w) - \psi(a)$.

- Suppose $q \in [q_0, q^1]$. $F(q|a)$-cdf, $f(q|a)$-pdf.

$PP(a, w(q)) = \int_{q0}^{q^1} V(q-w(q))f(q|a)dq \to_{a,w(q)}$ max.

s.t.$AG(a, w(q)) = \int_{q0}^{q^1} u(w(q))f(q|a)dq - \psi(a) \geq \bar{u}$

- If a unobservable, $a^* \in \arg\max_a AG(a, w(q))$.

FOC: (a): $\int_{q0}^{q^1} u(w(q))f_a(q|a)dq = \psi'(a)$.

Obtain: (**)

$$\mathcal{L} = \int \left[\begin{array}{c} h(w(q), a) + \lambda(u(w(q) - \psi(a))+ \\ \mu \left(u(w(q)) \frac{f_a(q|a)}{f(q|a)} - \psi'(a) \right) \end{array} \right] f(q|a)dq$$

Solution

$$\frac{V'(q-w(q))}{u'(w(q))} = \lambda + \mu \frac{f_a(q|a)}{f(q|a)}.$$

- Two problems:
  - FOC: (a): (**) is necessary (internal!) not sufficient (!).
  - (**) $\frac{f_a(q|a)}{f(q|a)}$ determine which qs are rewarded (punished)
- Ex 4. Suppose $a^* = a_1 > a_0$ (only two values of a).

$$\frac{V'(q-w(q))}{u'(w(q))} = \lambda + \mu \left[1 - \frac{f(q|a_0)}{f(q|a_1)}\right]$$

$\frac{f(q|a_0)}{f(q|a_1)} > 1$ (< 1) determines q is punished (rewarded).

3.4 On supports and distributions

- Principal and agent are risk-neutral:

  If for all a, $F(q|a)$ are linearly independent (finite number of output levels), first-best is obtainable: a la Cremer & McLean. (same critique)

- Suppose $q = a + \varepsilon$, $\varepsilon \in [-x, x]$. Again, first-best is achievable (risk-aversion is fine, no limited liability), with severe penalties for $q < a^* - x$.

  Same might be true (in approx) even if ε is unbounded, and agent is risk-averse.

- HYP: Subcase (extends for finite a's: $a_L = 0$; $a_H = 1$. If q^0 exists such that $\frac{f(q|a_H)}{f(q|a_L)} \to 0$ when $q \to q^0$ first-best can be approximated

3.5 Grossman & Hart approach

Finite # of q's: $0 \leq q_1 < q_2 < \cdots < q_n$.

$p_i(a) = \Pr(q = q_i | a)$

$V = q - w$

$U(w, a) = \phi(a) u(w(q)) - \psi(a)$,

u with usual properties, and $lim_{w \to w_o +} u(w) = -\infty$.

- a observable:

$V = \sum_{i=1}^{n} p_i(a)(q_i - w_i) \to_{a, w(q)} max$

s.t. $U(w, a) \geq \bar{u}$.

Full insurance, thus $w = u^{-1}\left(\frac{\bar{u}+\psi(a)}{\phi(a)}\right)$

then $V = \sum p_i(a) q_i - w \to_a$ max.

- a unobservable: $V \to_{w(q)}$ max, s.t $U(w, a) \geq \bar{u}$,

(IC) $U(w, a) = \max_{\hat{a} \in A} U(w, \hat{a})$.

To solve: two-step program:

1. Find how to implement given a in the least-costly way, optimize over i.

2. Optimize minimal costs over a.

- Trick, write down the whole problem as a convex problem (so SOC automatically satisfied). Instead of searching over w_i, search over $u_i = u(w_i)$.

Let $\mathcal{U} = \operatorname{Im} u(w_0, \infty)$. Assume: $\frac{\bar{u}+\psi(a)}{\phi(a)} \in \mathcal{U}$.

Define $w_i = h(u_i) = u^{-1} u(w_i)$.

Thus, the program: *given a.

$\min_{u_1,\ldots,u_n} \sum_{i=1}^{n} p_i(a)h(u_i)$

s.t.

$$\begin{cases} \sum_{i=1}^{n} p_i(a)(\phi(a)u_i - \psi(a)) \geq \bar{u}, \\ \sum_{i=1}^{n} p_i(a)(\phi(a)u_i - \psi(a)) \geq \\ \sum_{i=1}^{n} p_i(\hat{a})(\phi(\hat{a})u_i - \psi(\hat{a})), \text{ for all } \hat{a} \in A \end{cases}$$

Note: Linear constraints, convex objective.

Define $C(a) = \inf \left\{\sum_{i=1}^{n} p_i(a)h(u_i) | u \text{ implements } a\right\}$.

If, no u exists, define $C(a) = \infty$.

It is important that $p_i(a) > 0$ for all i and a. If $C(a) < \infty$, then each w_i less ∞.

Step 2. $\max_{a \in A} \left\{\sum_{i=1}^{n} p_i(a)q_i - C(a)\right\}$.