14.126 (Game Theory) Final Examination

Instructions: This is an open-book exam – you may consult written material but you may not consult other humans. There are four questions, weighted equally. You have 24 hours to complete the exam from the time you first open the envelope.

Please begin your answer to each question on a new page.

1. This question is about forward induction and knowledge. We have two players, 1 and 2. First, player 1 decides between staying out and entering. If he enters, then they play a battle of the sexes game: the players simultaneously choose between L and R, and the payoffs are (3,1) if both players choose L, (1,3) if both players choose R, and (0,0) otherwise. If player 1 stays out, the game ends with payoff vector $(v, 2)$, which is known by player 1. Everything described so far is common knowledge. We maintain the following assumptions, which we will refer to as forward induction.

   (i) Player 1 is sequentially rational.

   (ii) Player 2 is sequentially rational.

   (iii) At each information set he moves, player 2 is certain that player 1 is sequentially rational.

   (iv) At each information set he moves, player 1 is certain about (ii) and (iii).

   1. Assume that it is common knowledge that $v = 2$. Which strategies are consistent with forward induction?
   2. Now, v can be either 2 or -1. Assume that it is common knowledge that, for some small probability $\epsilon > 0$, player 2 believes (before observing player 1's move) that $v = 2$ with probability $1 - \epsilon$ and $v = -1$ with probability ϵ. [In other words, Nature chooses between 2 and -1 with

probabilities $1-\epsilon$ and ϵ; player 1 knows Nature's move, but player 2 does not.] Which strategies are consistent with forward induction?

3. Now, we will consider the case that player 1 does not know whether player 2 knows v. We have four states HK, HN, LK, and LN with probabilities $(1-\epsilon)^2$, $(1-\epsilon)\,\epsilon$, $(1-\epsilon)\,\epsilon$, and ϵ^2, respectively, for some small probability $\epsilon > 0$. We have $v = 2$ when the first letter is H, and $v = -1$ when the first letter is L. The information partitions of players 1 and 2 are {{HK,HN},{LK,LN}} and {{HK},{LK},{HN,LN}}. Show that, for sufficiently small ϵ, at any strategy consistent with forward induction player 1 enters independent of v. [Note that players do not get any exogenous information about the states during the game.]

4. Comparing your answers to the previous parts (especially (1) and (2)), briefly discuss the modeling assumptions behind the forward induction arguments. Discuss also the sensitivity of the forward induction arguments with respect to alternative ways to model knowledge.

2. This question aims to illustrate how we can use hyperbolic discounting to express the players' intertemporal trade-offs in equilibrium. Consider the Rubinstein's alternating-offer bargaining model, where two risk-neutral players try to divide a dollar. Think of each player as an infinite sequence of players, one at each date. In this model, the players use exponential discounting, so that the payoff for a player i at any time t from an agreement at time $s \geq t$ that gives x to i is $\delta_i^{t-s}x$. We will now allow more general discounting functions so that the above payoff is any $\delta_{i,t,s}x$ where $\{\delta_{i,t,s}\}_{s=t}^{\infty}$ is a strictly decreasing sequence of numbers in $(0,1]$ with $\delta_{i,t,t} = 1$ and $\lim_{s\to\infty} \delta_{i,t,s} = 0$.

   1. First assume that $\delta_{i,t,s}$ only depends on i and $s-t$. Find a subgame-perfect equilibrium. Which terms enter into the expression for the equilibrium division?

   2. Now compute a subgame-perfect equilibrium for general case described in the question. Which terms enter into the expression for the equilibrium division?

      [Hint: Write the continuation value of the proposer at time t in terms of his continuation value at time $t+2$. The solution for a difference equation $X_n = a_n + b_{n+1}X_{n+1}$ is given by $X_n = a_n + \sum_{k=n+1}^{\infty}\left(\prod_{l=n+1}^{k} b_l\right)a_k$

under the assumption $\lim_{k\to\infty}\left(\prod_{l=n+1}^{k} b_l\right) X_k = 0$, which is satified by your equation.]

3. Which intertemporal trade-offs play a role in equilibrium?

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