14.126 GAME THEORY

PROBLEM SET 2

MIHAI MANEA

Due by Wednesday, March 31, 5:00 PM

Question 1

Find all (a) Nash, (b) trembling-hand perfect, (c) proper equilibria (in pure or mixed strategies) of the following normal-form game.

	L	R
U	2,2	2,2
M	3,3	1,0
D	0,0	1,1

Question 2

Apply the forward-induction iterative elimination procedure described below to the following game. We have two players, 1 and 2, who will play the Battle of the Sexes (BoS) game with payoff matrix shown below:

	A	B
A	3,1	ε, ε
B	ε, ε	1,3

where ε is a small but positive number.

Before playing this game, first, player 1 decides whether or not to burn a util; if so, his payoffs will decrease by 1 at each strategy profile of BoS. Then, after observing whether 1 has burned a util, player 2 decides whether or not to burn a util, in which case her payoffs will decrease by 1 at each strategy profile of BoS. Then, after both players have observed each other's decisions to burn or not, they play BoS.

The iterative procedure is as follows. Let S_i be player i's pure strategy space. A *belief system* μ_i for player i specifies, for each information set h of player i, a probability distribution $\mu_i(h)$ over S_{-i}.

- Let $S_i^0 = S_i$.
- At any step t, for each player i, let Δ_i^t be the set of all belief systems μ_i with the following property:

  for any information set h of i that can be reached by some strategy profile in $S_i \times S_{-i}^t$, if $\mu_i(s_{-i}|h) > 0$, then $s_{-i} \in S_{-i}^t$ and h is reached by (s_i, s_{-i}) for some $s_i \in S_i$.

  For each player i, and each pure strategy $s_i \in S_i^t$, eliminate s_i if there does not exist any $\mu_i \in \Delta_i^t$ such that s_i is sequentially rational with respect to μ_i. Let S_i^{t+1} be the set of remaining strategies.
- Iterate this until the elimination stops.

(If you have done this correctly, you should find in the end that the procedure predicts the outcome that is best for player 2.)

Question 3

(a) Consider the repeated game $RG(\delta)$, where the stage game is matching pennies:

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

For any discount factor $\delta \in (0, 1)$, find all the subgame-perfect equilibria of the repeated game.

(b) A game $G = (N, A, u)$ is said to be a *zero-sum game* if $\sum_{i \in N} u_i(a) = \sum_{i \in N} u_i(a')$ for all $a, a' \in A$. For any discount factor $\delta \in (0, 1)$ and any two-player zero-sum game, compute the set of all payoff vectors that can occur in an SPE of the repeated game $RG(\delta)$.

Question 4

Consider the three-player coordination game shown below.

	A	B
A	1,1,1	0,0,0
B	0,0,0	0,0,0

A

	A	B
A	0,0,0	0,0,0
B	0,0,0	1,1,1

B

Show that each player's minmax payoff is 0, but that there is $\varepsilon > 0$ such that in every SPE of the repeated game $RG(\delta)$, regardless of the discount factor δ, every player's payoff is at least ε. Why does this example not violate the Fudenberg-Maskin folk theorem?

Question 5

Consider a repeated game with imperfect public monitoring. Assume that the action space and signal space are finite. Let $E(\delta)$ be the set of expected payoff vectors that can be achieved in perfect public equilibrium, where public randomization is available each period. Show that if $\delta < \delta'$, then $E(\delta) \subseteq E(\delta')$.

Question 6

(a) Let $G = (N, A, u)$ be a *finite* normal-form game. Consider the repeated game $RG(\delta)$ for some $\delta \in (0, 1)$. Mixed actions are not observed; only their realizations are. The set of (behavioral) strategy profiles is endowed with the product topology: a sequence of strategy profiles (σ^k) converges to σ if and only if $\sigma_i^k(h) \to \sigma_i(h)$ for every player i and history h.

Show that with this topology, the set of all (mixed) SPE profiles is closed.

(b) Let $G = (N, A, u)$ be a normal-form game, where $A_i = [0, 1]$ and the payoff function u_i is continuous on A, for each player i. The set of pure-strategy profiles is endowed with the product topology as above.

Show by example that the set of pure-strategy SPE profiles is not necessarily closed.

14.126 Game Theory
Spring 2010