Problem Set 2 - Solutions

Question (i)

Part 1

Assume first that for every $P \in \Delta(S)$, P_X FOSD P_Y. Fix $s \in S$ and take P such that $P(s) = 1$. Since P_X FOSD P_Y, it must be that

$$u(X(s)) = E_P[u(X)] \geqslant E_P[u(Y)] = u(Y(s))$$

for all $u : \mathbb{R} \to \mathbb{R}$ increasing. Therefore, by taking u to be the identity function, we get $X(s) \geqslant Y(s)$. Since the choice of s was arbitrary, we obtain that $X \geqslant Y$, as wanted.

Assume now that $X \geqslant Y$. Fix $P \in \Delta(S)$ and $u : \mathbb{R} \to \mathbb{R}$ increasing. Since $X \geqslant Y$, we have that $u(X) \geqslant u(Y)$. By monotonicity of the expectation $E_P[u(X)] \geqslant E_P[u(Y)]$. Since the choice of P and u was arbitrary, we get that P_X FOSD P_Y for all $P \in \Delta(S)$, as wanted.

Part 2

Let $S = \{1, 2\}$, $X(s) = s$, while $Y(1) = 2$ and $Y(2) = 1$. Pick $P \in \Delta(S)$ such that $P(2) = 1$. Then P_X FOSD P_Y but $X \ngeqslant Y$.

Question (ii)

See solution to Question 2 from 2014 pset2. Aside: the statement of this question should be interpreted as follows: Find $u \in \mathcal{U}$ and $G \in \mathbb{R}$ to minimize G subject to $\frac{1}{2}u(w_0+G)+\frac{1}{2}u(w_0-L) \geqslant u(w_0)$. In other words, you are free to choose **both** u and G.

Question (iii)

See solution to Question 1 from 2014 pset3.

Question (iv)

See solution to Question 3 from 2014 pset3.

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