Handout on contingent commodities in an exchange economy

Consumer choice

$$\begin{aligned} &\text{Max} \sum_s \pi_s^h u^h\left(x_s^h\right) \\ &\text{s.t.} \quad \sum_s p_s x_s^h = \sum_s p_s e_s^h \end{aligned} \tag{1}$$

FOC

$$\pi_s^h u'^h\left(x_s^h\right) = \lambda^h p_s \tag{2}$$

Market clearance

$$\sum_h x_s^h = \sum_h e_s^h \tag{3}$$

I. Everyone the same: $\pi_s^h = \pi_s;\; u^h(x) = u(x);\; e_s^h = e_s$

$$\begin{aligned} \lambda p_s &= \pi_s u'(e_s) \\ \lambda p(s) &= \pi(s) u'(e(s)) \end{aligned} \tag{4}$$

Logarithmic derivative

$$\frac{p'(s)}{p(s)} = \frac{\pi'(s)}{\pi(s)} + \frac{u''(e(s))}{u'(e(s))} e'(s) \tag{5}$$

II. Same probabilities, different utilities: $\pi_s^h = \pi_s$

$$\frac{u''^h(x_s^h)}{u'^h(x_s^h)} x'^h(s) = \frac{p'(s)}{p(s)} - \frac{\pi'(s)}{\pi(s)} \tag{6}$$

III. Same utility (logarithmic), different probabilities, no aggregate variation

FOC

$$\pi_s^h u'\left(x_s^h\right) = \lambda^h p_s \tag{7}$$

$$\pi_s^h / x_s^h = \lambda^h p_s \tag{8}$$

Market clearance

$$\sum_h x_s^h = \sum_h e_s^h = E \tag{9}$$

IV. General utility (Yaari) $u^h\left(x_1, ..., x_S\right)$

$$\frac{\partial u^h / \partial x_s^h}{\partial u^h / \partial x_{s'}^h} = \frac{\partial u^{h'} / \partial x_s^{h'}}{\partial u^{h'} / \partial x_{s'}^{h'}} \tag{10}$$