**Answer all three questions. They all count equally.**

1.      **Part A.**  Consider a two-person (Ann and Bob) two-good (x and y) competitive exchange

economy.  Ann has utility function

$$U^A = 4x^A + 2y^A$$

Bob has utility function

$$U^B = Min[3x^B, 3y^B]$$

Both have the nonnegative quadrant as a consumption possibility set.  Initial endowments

are: Ann: 2 units of $x$ and 6 units of $y$; and Bob: 10 units of $x$ and 18 units of $y$.

In answering the following questions, provide brief explanations, including relevant

definitions, as to why your answer is correct.  You may use algebra <u>or</u> geometry.

a.      Describe the set of Pareto Optima.  Be precise in your definition of Pareto optimality
       and careful about corner solutions.  If different definitions give different Pareto
       optimal sets, describe them.

b.      Describe the core.  Be precise in your definition of the core.

c.      Describe the prices and quantities in the two-person competitive equilibrium.  Is
       there more than one price ratio that supports competitive equilibrium?  Explain.

**Part B.**  Now assume that a third person is also in the economy, Charlie, who has the

nonnegative quadrant as a consumption possibility set and preferences

$$U^C = 2x^C + y^C$$

Now assume that initial endowments are: Ann: 1 unit of $x$ and 3 units of $y$; Bob: 10 units

of $x$ and 18 units of $y$; Charlie: 1 unit of $x$ and 3 units of $y$.

d.      Describe the prices and quantities in the three-person competitive equilibrium.  Is
       there more than one price ratio that supports competitive equilibrium?  Explain.

2.    Consider a two-good (apples, denoted $x$, and applesauce, denoted $y$) competitive economy with

two agents, Ann and Bob and one firm. Ann has utility function

$$U^A = 4x^A + 4\log\left[y^A\right].$$

Bob, who is affected by the applesauce eaten (noisily) by Ann, has utility function

$$U^B = 6x^B + 6\log\left[y^B\right] - 2y^A.$$

Both have the nonnegative quadrant as the consumption possibility set and each has an initial

endowment of 4 apples.

There is a single competitive firm that can convert apples into applesauce. If it uses $x^f$ apples, it

produces $5x^f$ units of applesauce. The firm is owned by Ann.

a.  Determine competitive equilibrium prices and quantities. Is equilibrium Pareto optimal?
    Explain.

b.  Describe the complete set of tax (and lump-sum transfer) policies that can maximize a weighted
    sum of the utilities of Ann and Bob (starting from initial endowments) with the property that
    consumption of both goods by both consumers is strictly positive. Derive the quantities in these
    equilibria. Are there any Pareto optima with the property that consumption of both goods by
    both consumers is strictly positive that are not in the allocations you have derived?

c.  Describe the complete set of tax (and lump-sum transfer) policies that can achieve a Pareto
    optimum (starting from initial endowments) with the property that it maximizes $U^A + U^B$.
    Derive the quantities in these equilibria. Explain intuitively the difference or lack of difference
    between the levels of $y^A$ in your answers to b. and c. Also explain intuitively the difference
    or lack of difference between the levels of $y^B$ in your answers to b. and c.

3.      Consider a one-period competitive economy with a complete set of contingent commodity

markets.  There are two states of nature.  There is one input good, $k$, which must be

allocated before the state of nature is known.  There are three consumer goods in each state

of nature, $x$, $y$, and $z$.   There is one consumer, Ann, who is an expected utility

maximizer, using subjective probabilities.

Ann believes the probabilities of states one and two are 1/4 and 3/4 respectively.  Ann's

utility function (for expected utility maximization) is:

$$U^A = 4x^A + 4y^A + 4\log\left[z^A\right].$$

Ann's consumption possibility set is the six dimensional space with nonnegative quantities

of the consumer goods in each state of nature.  Ann has an initial endowment of 12 units of

input. Ann owns all five firms that exist in the economy.

For each unit of input used by firm 1, there are 4 units of good $x$ produced if state 1 occurs

and 4 units of good $x$ produced if state 2 occurs.

For each unit of input used by firm 2, there are 4 units of good $y$ produced if state 1 occurs

and nothing if state 2 occurs.

For each unit of input used by firm 3, there are 4 units of good $y$ produced if state 2 occurs

and nothing if state 1 occurs.

For each unit of input used by firm 4, there are 2 units of good $z$ produced if state 1 occurs

and nothing if state 2 occurs.

If firm 5 uses $k$ units of input, $k \geq 1/16$, then $(k-1/16)^{1/2}$ units of good $z$ are produced if

state 2 occurs and nothing if state 1 occurs.

Does there exist a complete market competitive equilibrium?  If so, derive the competitive

equilibrium prices and quantities.  If not, show why not.  Be sure to show the derivation of

equations that describe equilibrium.  Be careful about corner conditions.