PROBLEM SET #3

Due: Tuesday February 22

1.   Consider a two-person (Ann and Bob) two-good (x and y) competitive exchange economy. Ann has utility function

$$U^A = 4x^A + 4y^A$$

Bob has utility function

$$U^B = Min[2x^B, 2y^B]$$

Both have the nonnegative quadrant as a consumption possibility set.

In answering the following questions, provide brief explanations, including relevant definitions, as to why your answer is correct. You may use algebra or geometry.

Ann's initial endowment is 2 units of x and 10 units of y; and Bob's initial endowment is 10 units of x and 14 units of y.

     a.   Describe the set of Pareto Optima. Be precise in your definition of Pareto optimality and careful about corner solutions. If different definitions give different Pareto optimal sets, describe them.

     b.   Describe the core. Be precise in your definition of the core.

     c.   Describe the core in the four-person economy made up of two Anns and two Bobs, each Ann like Ann above and each Bob like Bob above.

2.   Consider a two-person (Ann and Bob) two-good (x and y) competitive exchange economy. Ann has utility function

$$U^A = Min \{x, 2y\}.$$

Bob has utility function

$$U^B = Min \{3x, y\}.$$

Both have the nonnegative quadrant as a consumption possibility set.

The aggregate endowment in this economy is (12,12).

(a) Contrast the sets of Pareto Optimal allocations under the two different definitions of Pareto Optimality given in class – with weak and strong inequalities on utilities.

(b) Assume that Ann has all of good x and Bob has all of good y. Describe the competitive equilibrium.

(c) Describe the set of individual endowments, consistent with the aggregate endowment, that result in a competitive equilibrium with positive prices for both goods.

3. A two-good exchange economy has equal numbers of two types of consumers. Type A has initial endowment (1,0) and utility function

$$ax_1 + bx_2.$$

Type B has initial endowment (0,1) and utility function

$$\text{Min } \{cx_1, dx_2\}.$$

$$(a, b, c, d > 0.)$$

(a) Determine the competitive equilibrium prices and quantities. (Are prices unique? Are quantities unique?)

(b) Calculate the core of this economy if there is only one consumer of each type.

(c) Assume a=b and c=d. Assuming that there are at least two individuals of each type, show that the competitive equilibria are the only members of the core.

4. Does an exchange economy with identical consumers with strictly monotonic, strictly convex preferences have unique competitive equilibrium prices? Are prices unique if the quantities are interior to the consumption possibility set?

5. Consider the following economy: There are three goods, legume, tillip and quillip (L, T,

and Q), two consumers (called 1 and 2), and two firms (called x and y). Firm x is owned entirely by consumer 1 and makes tillip out of legume according to the simple linear production technology $T \le 3L$. That is, for every unit of legume input, this firm produces three times as many (or less) units of tillip. Firm y is owned entirely by consumer 2 and makes quillip out of legume according to the production technology $Q = 4L$. Each consumer initially owns 5 units of legume. Consumer 1 has utility function $u_1(T,Q) = 6 + .4\ln(T) + .6\ln(Q)$. Consumer 2 has utility function $u_2(T,Q) = 8 + 1n(T) + 1n(Q)$.

(a)   What is the general equilibrium of this economy? Assume that firms take prices as given and are profit maximizers, and consumers take prices as given. When you give prices, normalize them so the price of legume is $1. What would be the general equilibrium if the shareholdings were reversed? If each consumer held a half-share in each firm?

(b)   What is the set of all feasible, Pareto efficient allocations for this economy? (Kreps, 8.11)

6.   Consider a competitive economy with two consumers, Ann and Bob, and one firm. There are 2 goods in the economy, labor, x, and consumption, y (both measured positively, i.e. not the Debreu sign convention). Each consumer has 18 units of labor which are supplied inelastically and utility function log[y]. The production function of the firm is

$$y = x^{1/2}.$$

(a)   Find competitive equilibrium prices and quantities assuming Ann owns the firm.

(b)   Find competitive equilibrium prices and quantities assuming Bob owns the firm.

Now assume the production function of the firm is $y = x/6$.

(c)   Find competitive equilibrium prices and quantities assuming Ann owns the firm.

(d)   Find competitive equilibrium prices and quantities assuming Bob owns the firm.

7.   Consider a two-good (apples, denoted x, and applesauce, denoted y, both measured positively) competitive economy with two agents, Ann and Bob and one firm. Ann has utility function

$$U^A = 2\log[1+x^A] + 3y^A.$$

Bob has utility function

$$U^B = \log[1+x^B] + 3y^B.$$

Both have the nonnegative quadrant as the consumption possibility set and each has an initial endowment of 4 apples.

The firm can convert apples into applesauce. If it uses x^f apples (with $x^f > k$), it produces $(x^f - k)^{1/2}$ units of applesauce. The firm is owned by Ann.

Assume k=2.

Find competitive prices and quantities.

8.   Consider a two-good (apples, denoted x, and applesauce, denoted y, both measured positively) competitive economy with two agents, Ann and Bob and one firm. Ann has utility function

$$U^A = 2\log[1+x^A] + 3y^A.$$

Bob has utility function

$$U^B = \log[1+x^B] + 3y^B.$$

Both have the nonnegative quadrant as the consumption possibility set and each has an initial endowment of 4 apples.

The firm can convert apples into applesauce. If it uses x^f apples (with $x^f > k$), it produces $(x^f - k)^{1/2}$ units of applesauce. The firm is owned by Ann. Consider values of k>8, so that the economy is <u>not</u> capable of producing applesauce.

Give your answers as functions of k, where appropriate.

(a)   For what prices is there zero demand for applesauce?

(b)   For what prices does the firm (maximizing profits as a price-taker) prefer to have zero production?

(c)   For what values of k (k>8) does a competitive equilibrium exist? For these values of k, determine competitive equilibrium prices and quantities.