PROBLEM SET #2

Due: Tuesday, February 15, 2005

1. Consider the following social choice problem in the setting of consumption of two goods by two consumers. The two goods are called tillip and quillip and the two consumers are called 1 and 2. Consumer 1 has utility function $U_1(t,q) = 6 + .4 \ln(t) + .6 \ln(q)$ (where t is the amount of tillip 1 consumes and q is the amount of quillip). Consumer 2 has utility function $U_2(t,q) = 8 + \ln(t) + \ln(q)$. The social endowment consists of 15 units of tillip and 20 units of quillip.

   What is the set of all feasible, Pareto efficient allocations of the consumption good for this society? (Kreps, 5.2b.)

2. In a perfectly competitive pure exchange economy, there are two types of persons, denoted by A and B, with equal numbers of each. They consume goods X and Y, with utility functions:

   $U^A = X - 100\, e^{-Y/10} + 47$ for type A
   $U^B = Y - 100\, e^{-X/10} + 100$ for type B

   and they have endowments of 40 of X for type A and 50 of Y for type B.

   (i) Writing p for the relative price of Y, derive the demand functions of the two groups, and comment on their properties. (Watch for corner solutions.)

   (ii) Graphically, or otherwise, show that there are multiple competitive equilibria. (Hint: consider the local behavior of excess demand at p=1.)

3. There are two goods X and Y and two consumers A and B with the following utility functions and endowments:

   $U^A = a \ln X^A + (1-a) \ln Y^A$, $E^A = (0,1)$, $0<a<1$.
   $U^B = \min (X^B,Y^B)$, $E^B = (1,0)$.

   Calculate the competitive equilibrium prices and allocation. (Varian 17.4)

4. We have two agents with <u>indirect</u> utility functions:

$$v^A(p_1,p_2,y^A) = \ln y^A - a \ln p_1 - (1-a) \ln p_2$$
$$v^B(p_1,p_2,y^B) = \ln y^B - a \ln p_1 - (1-a) \ln p_2$$

and initial endowments

$$E^A = (1,1),\ E^B = (1,1).$$

Calculate the competitive market clearing prices. (Varian 17.6)

5. Consider an economy with 15 consumers and 2 goods. Consumer 3 has a Cobb-Douglas utility function $U^3 = \ln X + \ln Y$. At a certain Pareto efficient allocation, consumer 3 has an allocation (10,5). What are the competitive prices that support this Pareto optimal allocation? (Varian 17.9.)

6. Either prove or find a counterexample to the following proposition:

In an economy with identical consumers, an increase in the endowment of one good will lead to a reduction in the price of that good relative to all other goods.

7. Consider a competitive exchange economy with a continuum of consumers. There is a unit measure of these consumers, indexed by j, with j being uniformly distributed in the interval between 1 and 2. Consumer j has utility function

$$U^j = x + jy$$

and initial endowment $E^j = ((1+j)^{-2}, (1+j)^{-2})$.

Describe the competitive equilibrium prices and allocation.