14.121 Problem Set #2

Due September 28, 2005

1. Suppose a consumer satisfies the von Neumann-Morgenstern axioms and is (strictly) risk averse. She initially has monetary wealth w. There is some probability p that she will lose an amount L. The consumer can purchase insurance that will pay her q dollars in the event that she incurs this loss. The amount she has to pay for this insurance is πq; here π is the premium per dollar of coverage.

(a) How much insurance will the consumer purchase? That is, what is the choice of q (as a function of the parameters w, p, L and π) that maximizes her expected utility? (Note: It's fine to just give an equation that implicitly defines q.)

(b) In the special case where the insurance is *actuarially fair* (that is, $\pi = p$), how much insurance will the consumer purchase? What about if $\pi > p$? Discuss.

2. Consider a town somewhere not very far from MIT which is filled with utility-maximizing citizens, all of whom have preferences which satisfy the von Neumann-Morgenstern axioms, which can be represented by a utility function $u(w)$, where the function u is the same for all agents. Assume that this utility function is differentiable as many times as you like, and further assume it is strictly increasing and concave.

Every individual has some initial wealth as well as a car which must be parked at metered parking spaces each day. The cost of the meter is given by m. If the meter is not paid, it indicates "violation." The town hires police who patrol the area, and with probability p the violation is spotted and a fine f is levied. If the police do not spot the violation, the individual pays nothing.

Consider the problem faced by an individual who maximizes their utility for the day by choosing whether or not to feed the meter (for non-Americans, "feed" is slang for "put money into"). Let Δ denote the returns to the agent from feeding the meter, that is, the difference between the agents expected utility from feeding the meter and their expected utility from parking illegally.

(a) Suppose that the town council considers funding more police, which increases p, versus raising the fine f. Compute the elasticity of Δ with respect to p, and compare it to the elasticity of Δ with respect to f. To which policy change are the citizens more responsive?

(b) For the remainder of this problem suppose that citizens of this town differ according to their initial wealth, w_0, on the range $[\underline{w}, \bar{w}]$. Suppose also that the parameters of the problem are such that Δ is decreasing in initial wealth, and that we observe some citizens feeding the meters and others parking illegally.

Write a simple expression which determines which group of people feeds meters and which group parks illegally. Show that the assumption that the both groups are nonempty requires assuming that the price of the meter, m, is larger than the expected value of the fine, pf, from parking illegally?

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(c) Qualitatively, how does the set of people who feed the meters change with f, m and p? Prove your answer and interpret your results.

3. MWG question 6.B.5

4. The standard financial advice is that younger people should invest a higher fraction of their wealth in stocks than older people. Discuss why this might (or might not) make sense.