Lecture 8
Backward Induction

14.12 Game Theory

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Road Map

1. Backward Induction
2. Examples
3. Application: Stackelberg Duopoly
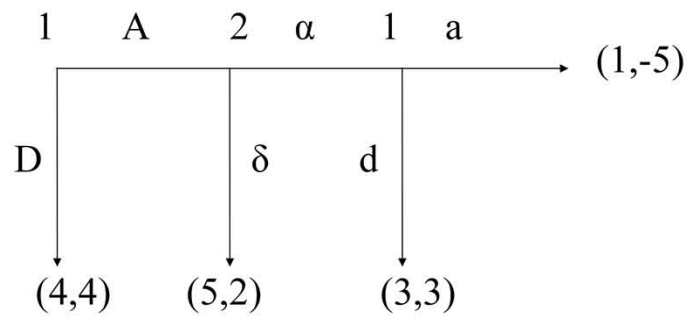4. [Next Application: Negotiation]

Definitions

Perfect-Information game is a game in which all the information sets are singleton.

Sequential Rationality: A player is sequentially rational iff, at each node he is to move, he maximizes his expected utility conditional on that he is at the node – even if this node is precluded by his own strategy.
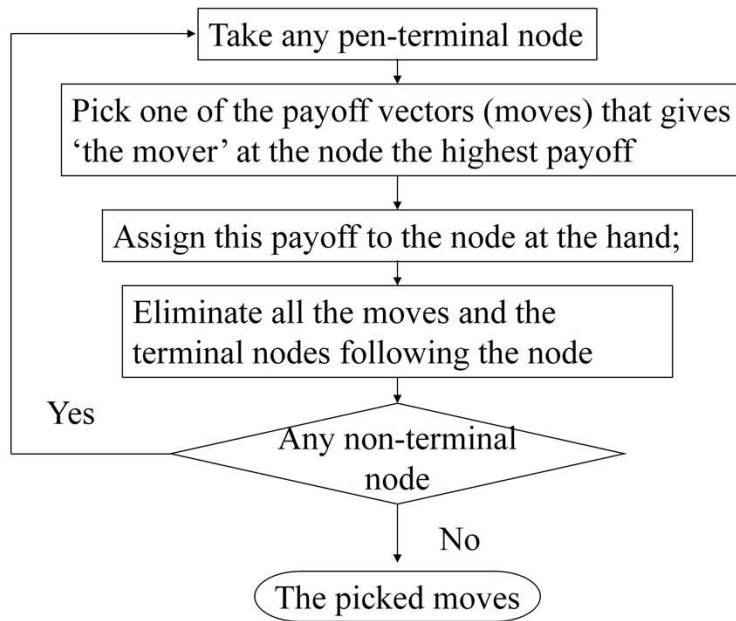
Backward Induction: Apply sequential rationality and the "common knowledge" of it as much as possible (in finite games of perfect information).
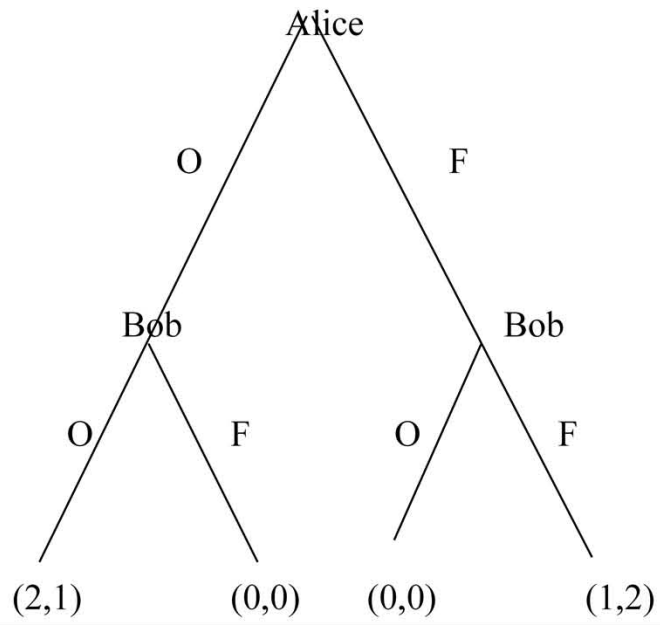
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A game

```
1      A      2    α    1    a
├──────────────┬─────────┬────────────→ (1,-5)
│              │         │
D              δ         d
│              │         │
↓              ↓         ↓
(4,4)        (5,2)     (3,3)
```
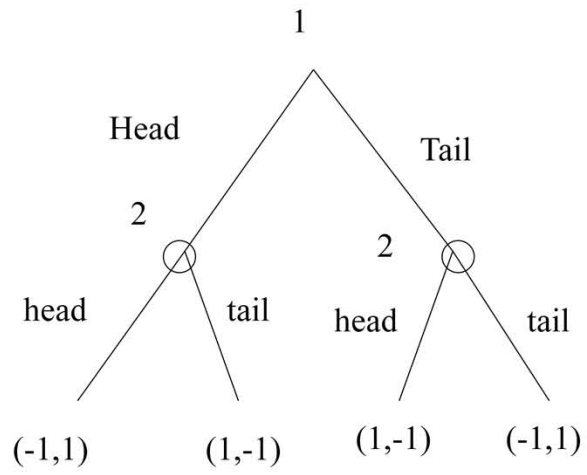
Backward Induction

Take any pen-terminal node

↓

Pick one of the payoff vectors (moves) that gives 'the mover' at the node the highest payoff

↓

Assign this payoff to the node at the hand;

↓

Eliminate all the moves and the terminal nodes following the node

↓

Any non-terminal node

Yes

No

The picked moves

Battle of The Sexes with perfect information

Alice

O   F

Bob   Bob

O   F   O   F
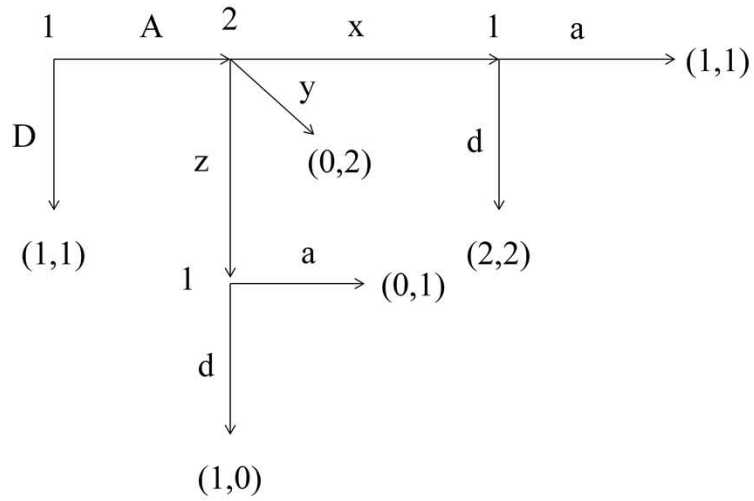
(2,1)   (0,0)   (0,0)   (1,2)

Note

- Backward Induction always yields a Nash Equilibrium.

- There are Nash equilibria that are different from the Backward Induction outcome.

- Sequential rationality is stronger than rationality.
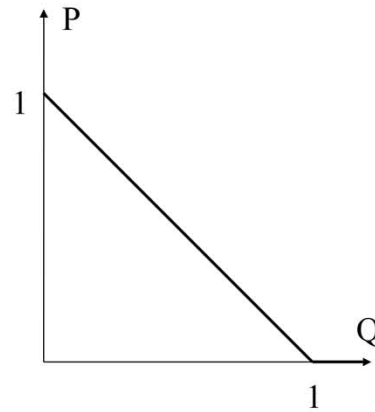
Matching Pennies (wpi)

A game with multiple solutions

1   A   2   x   1   a   → (1,1)

y → (0,2)

D

z

d → (2,2)

(1,1)

1   a → (0,1)

d

(1,0)

Stackelberg Duopoly

Game:

$N = \{1,2\}$ $c = 0$;

1. Firm 1 produces q_1 units
2. Observing q_1, Firm 2 produces q_2 units
3. Each sells the good at price
   $P = \max\{0, 1-(q_1+q_2)\}$.

$\pi_i(q_1, q_2) = q_i[1-(q_1+q_2)]$ if $q_1 + q_2 < 1$,

0   otherwise.

14.12 Economic Applications of Game Theory
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