Lecture 4
Dominance

14.12 Game Theory

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Road Map

1. Dominance & Rationality
2. Dominant-Strategy Equilibrium
3. 2^{nd} price auction

Prisoners' Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

Dominance

$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$

Definition: A pure strategy s_i* **strictly dominates** s_i if and only if

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i}.$$

A mixed strategy σ_i **strictly dominates** s_i iff

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \cdots + \sigma_i(s_{ik})u_i(s_{ik}, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i}$$

A rational player never plays a strictly dominated strategy.

Prisoners' Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

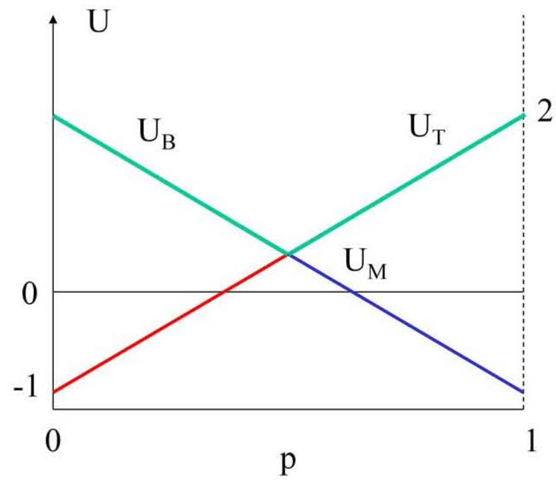
A Game

$U_M = 0$

$U_T = 2p-(1-p) = 3p-1$

$U_B = -p+2\ (1-p) = 2-3p$

	L	R
T	(2,0)	(-1,1)
M	(0,10)	(0,0)
B	(-1,-6)	(2,0)
	p	1-p



Weak Dominance

Definition: A pure strategy s_i* weakly **dominates** s_i if and only if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_{-i}.$$

and at least one of the inequalities is strict. A mixed strategy σ_i* **weakly dominates** s_i iff

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \cdots + \sigma_i(s_{ik})u_i(s_{ik}, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_i$$

and at least one of the inequalities is strict.

If a player is rational and cautious (i.e., he assigns positive probability to each of his opponents' strategies), then he will not play a weakly dominated strategy.

Dominant-strategy equilibrium

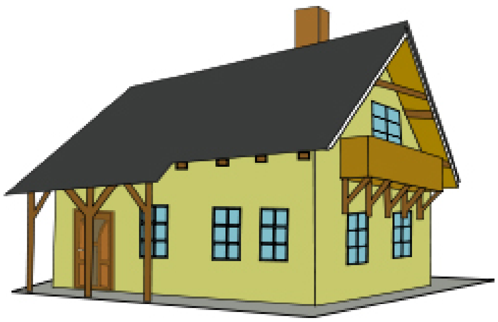
Definition: A strategy s_i* is a **dominant strategy** iff s_i* **weakly dominates** every other strategy s_i.

Definition: A strategy profile s* is a dominant-strategy equilibrium iff s_i* is a dominant strategy for each player i.

Prisoners' Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

Second-price auction



Courtesy of Machovka on OpenClipart.org.

- $N = \{1,2\}$ buyers;
- The value of the house for buyer i is v_i;
- Each buyer i simultaneously bids b_i;
- i* with $b_{i*} = \max b_i$ gets the house and pays the second highest bid

$$p = \max_{j \neq i} b_j.$$

2^{nd} price Auction

- Strategies:

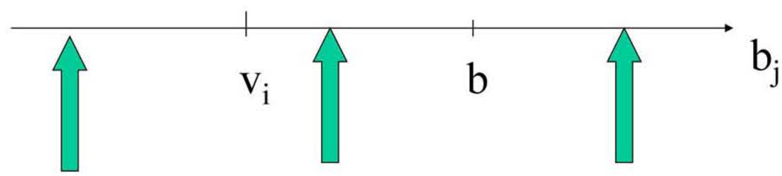
$$b_i \in [0,\infty)$$

- Payoffs:

$$u_i\,(b_i,b_j) = v_i - b_j \qquad \text{if } b_i > b_j$$
$$= (v_i - b_j)/2 \quad \text{if } b_i = b_j$$
$$= 0 \text{ otherwise.}$$

$b_i = v_i$ is a dominant strategy

$b_i = v_i$ dominates any $b > v_i$:



$u_i(v_i,b_j) = v_i - b_j$	$u_i(v_i,b_j) = 0$	$u_i(v_i,b_j) = 0$
$u_i(b,b_j) = v_i - b_j$	$u_i(b,b_j) = v_i - b_j < 0$	$u_i(b,b_j) = 0$

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