14.06 Final
Spring 2004

You have **1h30min**. Please answer each question in **separate booklets.** Each question counts for a total of 60 points. Allocate your time efficiently. If you can not answer some part analytically, but you can still give the intuition for the sought result, your answer will count partially.

GOOD LUCK!!!

Question 1   [60 points]

Consider an endogenous growth economy. Time is discrete, $t \in \{0, 1, ...\}$; there is no population growth; there is no exogenous technological change; and there is perfect competition. The source of growth is productive services provided by the government.

The representative firm maximizes profits,

$$\Pi_t = Y_t - r_t K_t - w_t L_t,$$

subject to the technology

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{1}$$

where $\alpha \in (0, 1)$ denotes the income share of capital and A_t denotes productivity in period t. We assume that the productivity of a firm increases with the level of public services. In particular, we denote the level of government spending per head by g_t and let

$$A_t = g_t^{1-\alpha}. \tag{2}$$

Note that the individual firm (or household) takes A_t as exogenous, but in general equilibrium A_t increases with g_t.

The government may impose a different tax on income from financial assets and a different tax on wages. We denote the tax on financial assets by τ^k and that on labor income by τ^w. The government budget writes

$$g_t = \tau^k r_t k_t + \tau^w w_t, \tag{3}$$

where $k_t = K_t/L_t$ is the capital per head (or per household). Accordingly, the budget of the representative household is

$$c_t + k_{t+1} = (1 - \tau^k) r_t k_t + (1 - \tau^w) w_t. \tag{4}$$

Preferences are given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad \text{where } u(c) = \ln c.$$

Finally, note that, from (2), (3), and (4), the resource constraint of the economy is

$$k_{t+1} = y_t - c_t - g_t. \tag{5}$$

(a) Characterize r_t and w_t from the firm's problem. Combine then with (1)-(3) to express g_t, y_t, r_t and w_t in terms of k_t and (τ^k, τ^w) *alone*. Similarly, write the resource constraint (5) in terms of (c_t, k_t, k_{t+1}) and (τ^k, τ^w) alone. Explain why this economy is an example of an $AK-$type economy. Explain how do τ^k and τ^w affect y_t and r_t.   [15 points]

(b) Write the Euler condition that characterizes optimal consumption growth in terms of (c_t, c_{t+1}) and (τ^k, τ^w) alone. How do τ^k and τ^w affect the incentives to save? Interpret. (Hint: Do not try to solve for savings. Another way to interpret this question is "how do τ^k and τ^w affect the rate of consumption growth?")   [10 points]

(c) Combining your results from parts (a) and (b) above, express the long-run growth rate of the economy $\gamma = c_{t+1}/c_t = y_{t+1}/y_t$ and the consumption-output ratio $c/y = c_t/y_t$ as functions of (τ^k, τ^w). [HINT: You should already have the long run growth rate of the economy from part (b). To solve for c/y , use your resource constraint from part (a) and divide through by k_t. You will then need to use the fact that on the balanced growth path, it must be that consumption and capital grow at the same rate in order to solve for c/y]   [5 points]

(d) Interpret the effects of τ^k and τ^w on γ and c/y.   [10 points]

(e) Fix $\tau^w = 0$ and let $\tau^k \in [0, 1]$. What value of τ^k maximizes c/y and what maximizes γ? Next, fix $\tau^k = 0$ and let $\tau^w \in [0, 1]$. What value of τ^w maximizes c/y and what maximizes γ? Why is there a difference?   [10 points]

(f) Suppose that the government can choose freely both τ^k and τ^w so as to maximize social welfare. Discuss what are the trade-offs the government faces in choosing the optimal tax rates that maximize social welfare. Is it optimal to use both $\tau^w > 0$ and $\tau^k > 0$, or just one of them, and if one which one? [10 points]

Question 2   [60 points]
Discuss whether each of the following statements is true, false, or uncertain. Provide a brief but clear explanation of your answer.

(a) If the aggregate technology exhibits constant returns with respect to the vector of accumulable factors (different types of capital), then the economy has necessarily a constant growth rate at all times, and it is impossible to make sense of conditional convergence.   [15 points]

(b) More competition necessarily promotes economic growth and social welfare, since firms are forced to produce more goods and extract less profits from consumers.   [15 points]

(c) Consider an individual agent. If her income varies randomly from one period to another, then her consumption will also vary from one period to another, but less so than her income.   [15 points]

(d) The neoclassical growth model (the RBC paradigm) can well account for the business-cycle variation in output, investment, employment, and total factor productivity.   [15 points]