14.05 Intermediate Macroeconomics
Problem set 1

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Problem 1.

Consider the growth model. Suppose that the total output produced at time t, Y_t, is governed by a Cobb-Douglas production function

$$Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha} \qquad 0 < \alpha < 1,$$

where K_t is the amount of capital employed in production in time t and L_t the amount of labor employed in production in time t.

Suppose that population is growing at a rate of $n > -1$, and the rate of capital depreciation is, $0 < \delta < 1$, and the saving rate is $0 < s < 1$.

a) Show that the Cobb-Douglas production function satisfies all the Neoclassical conditions

$$
\begin{aligned}
F(0, L_t) &= 0, & F(K_t, 0) &= 0, \\
F_L(K_t, L_t) &> 0 & F_K(K_t, L_t) &> 0, \\
F_{LL}(K_t, L_t) &< 0 & F_{KK}(K_t, L_t) &< 0,
\end{aligned}
$$

$$\lim_{L_t \to \infty} F_L(K_t, L_t) = 0$$

$$\lim_{L_t \to 0} F_L(K_t, L_t) = +\infty$$

and

$$\lim_{K_t \to \infty} F_K(K_t, L_t) = 0$$

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b) Find the output per worker at time t

$$y_t = \frac{Y_t}{L_t} = f(k_t)$$

where $k_t \equiv \frac{K_t}{L_t}$ is the capital-labor ratio at time t.

c) Show that the function $f(k_t)$ satisfy the Neoclassical properties since

$$f(0) = 0$$
$$f'(k_t) > 0$$
$$f''(k_t) < 0$$
$$\lim_{k_t \to 0} f'(k_t) = \infty$$
$$\lim_{k_t \to \infty} f'(k_t) = 0$$

d) Derive the law of motion of the economy in the Cobb-Douglas case. Note that in the general case the law of motion is

$$k_{t+1} = \frac{sf(k_t) + (1 - \delta)k_t}{1 + n} = \phi(k_t).$$

e) Draw the phase diagram in the plain (k_t, k_{t+1}).

f) What are the steady-state levels of k and y (i.e. what are $\bar{k}$ and $\bar{y}$)

g) Find the effects of an increase in the: saving rate s; rate of population growth n; on the steady state levels of the capital labor ration $\bar{k}$ (and therefore on income per capita $\bar{y}$). You can establish it, either geometrically or algebraically.

h) what is the effect of the initial level of the capital-labor ratio k_0 on the steady state levels of the capital labor ratio $\bar{k}$ and on $\bar{y}$. What is the economic significance of this result? What does it imply about convergence of income per capita across countries?

Problem 2

Are the effects that you find in part (g) of the previous problem consistent with the contemporary correlations across countries between (i) population growth and income per capita, (ii) saving rates and income per capita? To address this question, to go to the OECD website, obtain data on output per capita and on, saving rates and populations growth rates. Then, first draw the scatter plots, and then run the regressions of output per capita on saving rate and population growth rate. You can run the regressions in STATA or EXCEL. Reports the graphs and the coefficients.

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